Homework #1 Solutions

① $10^3(26^4)$

② a) $\binom{15}{3} = 455$

b) $15 \cdot 14 \cdot 13 = 2730$

③ a) $\binom{6}{2} = 15$

b) Through point C: $\binom{2}{1}\binom{4}{1} = 8$

Through point D: $\binom{4}{1}\binom{2}{1} = 8$

Through both C and D: $\binom{2}{1}\binom{2}{1} = 4$

Through C or through D: $8 + 8 - 4 = 12$

c) $15 - 12 = 3$

d) $(15-8) + (15-8) - 3 = 7 + 7 - 3 = 11$

④ $(2x-y)^6 = \binom{6}{0}(2x)^0(-y)^6 + \binom{6}{1}(2x)^1(-y)^5 + \binom{6}{2}(2x)^2(-y)^4 + \binom{6}{3}(2x)^3(-y)^3$

$+ \binom{6}{4}(2x)^4(-y)^2 + \binom{6}{5}(2x)^5(-y)^1 + \binom{6}{6}(2x)^6(-y)^0$

$= y^6 - 12xy^5 + 60x^2y^4 - 160x^3y^3 + 240x^4y^2 - 196x^5y + 64x^6$

⑤ For a k-element subset S of $N = \{1, 2, \ldots, n\}$, let i be the number of elements of S that are larger than the smallest element of S^c. There is 1 set S for which $i = 0$, namely $S = \{1, 2, \ldots, k\}$, and $1 = \binom{n-k+1}{0}$. If $i = 1$, then S consists of $1, 2, \ldots, k-1$ and one of the integers $k+1, \ldots, n$. Therefore there are $\binom{n-k}{1} = \binom{n-k-1+1}{1}$ such sets S. In general, S consists of $1, 2, \ldots, k-i$ and i of the integers $k-i+2, \ldots, n$. There are $\binom{n-k-1+i}{i}$ such S.

(6) a) $\frac{8!}{2!\,2!\,2!} = 7! = 5040$

b) $\frac{7!}{2!\,2!} = 1260$

c) $5040 - 1260 = 3780$

d) Starting with an A: $\frac{7!}{2!\,2!\,2!} = 630$, starting with an O: $\frac{7!}{2!\,2!} = 1260$

total: 1890

(7) EF : sum is even and the product is even, so both dice are even.

$E \cup F$: sum is even or the product is even, so $E \cup F$ is the whole sample space

FG^c: product is even and neither die is 1

EFG: both dice are even and at least one is 1, so this is $\emptyset$.

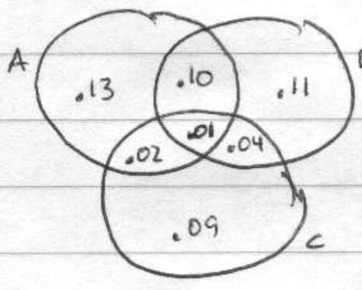
(8) There are 27 ordered triples with sum = 11, and 25 ordered triples with sum = 12 so sum = 11 is more likely.

(9) a) $.5 + .4 = .9$

b) $.5 + .4 = .9$

c) $.1$

(10)



a) $P(A \cup B \cup C) = .26 + .26 + .16 - .11 - .03 - .05 + .01 = .50$

b) $.10 + .02 + .01 + .04 = .17$

(11) $\dfrac{48(47)\cdots\cdots(40)(4)}{52(51)\cdots\cdots(43)} = \dfrac{42(41)(40)(4)}{52(51)(50)(49)} = \dfrac{275520}{6497400} \approx .0424$

(12) $\binom{52}{4} = 270725$

a) $\dfrac{\binom{4}{1}\binom{13}{4}}{\binom{52}{4}} = \dfrac{2860}{270725} \approx .011$

b) $\dfrac{\binom{13}{1}\binom{12}{2}\binom{4}{2}\binom{4}{1}\binom{4}{1}}{\binom{52}{4}} = \dfrac{82368}{270725} \approx .304$

c) $\dfrac{11(4)^4 - 11(4)}{\binom{52}{4}} = \dfrac{2772}{270725} \approx .01$

(13) P(more than one ball goes down at least one of the holes)

= 1 − P(all balls go through different holes)

$= 1 - \dfrac{\frac{7!}{2!}}{7^5} = 1 - \dfrac{7\cdot6\cdot5\cdot4\cdot3}{7\cdot7\cdot7\cdot7\cdot7} \approx 1 - .15 = .85$

(14) P(sum of 8) $= \frac{5}{36}$ P(product of 6) $= \frac{4}{36}$ in one roll.

Note that for one roll of two dice, the events "sum of 8" and "product of 6" are disjoint.

P(sum of 8 occurs 1st) $= \sum_{i=1}^{\infty}$ P(neither occurs on first $i-1$ rolls) P(sum of 8 on i^{th} roll)

$= \sum_{i=1}^{\infty} \left(\frac{27}{36}\right)^{i-1}\left(\frac{5}{36}\right) = \frac{5}{36}\left(\frac{1}{1-\frac{27}{36}}\right) = \frac{5}{9}$