

Math 213 Even Numbered Problems Answers

Section 12.1

#6: The circle on the plane $z = -2$ of radius 2 centered at $(0, 0, -2)$.

#12: The circle on the xz -plane of radius $\sqrt{3}$ centered at the origin.

#16: a. The circular disk on the xy -plane of radius 1 centered at the origin.

b. The circular disk on the plane $z = 3$ of radius 1 centered at $(0, 0, 3)$.

c. The solid circular cylinder of radius 1 with axis the z -axis.

#20: a. $x = 3$.

b. $y = -1$.

c. $z = 2$.

#24: $z = 1, (x + 3)^2 + (y - 4)^2 + (z - 1)^2 = 1$

#38: $\sqrt{3}$.

#46: $x^2 + (y + 1)^2 + (z - 5)^2 = 4$.

#54: a: $|z|$. b: $|x|$. c: $|y|$.

Section 12.2

#4: a: $\langle 5, -7 \rangle$. b: $\sqrt{74}$.

#20: $-2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$.

#26: 11, $11\mathbf{v}$ where $\mathbf{v} = \frac{9\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}}{11}$.

#34: $\sqrt{3}(-\mathbf{i} + \mathbf{j} + \mathbf{k})$

#40: $\langle -9, 0, 14 \rangle$.

#42: $\mathbf{u}_1 = -3\mathbf{v}, \mathbf{u}_2 = 7\mathbf{w}$

Section 12.3

#4: a. 13, 15, 3. b: $13/45$. c: $13/15$. d: $\frac{13}{225} \langle 2, 10, -11 \rangle$.

#10: $\arccos(2/3) \approx 0.841$ rad $\approx 48.2^\circ$.

#18: $\langle 1/2, 1/2, 0 \rangle + \langle -1/2, 1/2, 1 \rangle$.

#22: (went over in class)

#50: $\arccos(4)$

#56: There is an error in this problem. Change the equation $y = \sqrt{x}$ to $x = y^2$ (otherwise there will be just one point of intersection). With this change, the answers are: right angle at $(0, 0)$, $\arccos(4/5) \approx 36.87^\circ$ at $(1, -1)$.

Section 12.4

#2: $\mathbf{u} \times \mathbf{v}$: length: 5, direction: \mathbf{k} . $\mathbf{v} \times \mathbf{u}$: length: 5, direction: $-\mathbf{k}$.

#6: $\mathbf{u} \times \mathbf{v}$: length: 1, direction: \mathbf{j} . $\mathbf{v} \times \mathbf{u}$: length: 1, direction: $-\mathbf{j}$.

#24: a: $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are mutually perpendicular. b: \mathbf{r} and \mathbf{u} are parallel.

Section 12.5

#8: $x = 2 + 3t, y = 4 + 7t, z = 5 - 5t$.

#10: $x = 2 + t, y = 3 - 2t, z = t$.

#24: $x - 3y - z = 9$.

#26: $x - 2y + z = 6$.

#34: 3

#42: $8/3$

#62: $L1$ and $L3$ are parallel and not identical. $L1$ and $L2$ are skew and not intersecting. $L2$ and $L3$ are skew and intersects at $(1, 3, 2)$.

Section 13.1

#10: velocity: $\mathbf{i} + \sqrt{2}t\mathbf{j} + t^2\mathbf{k}$, acceleration: $\sqrt{2}\mathbf{j} + 2t\mathbf{k}$. At $t = 1$, speed = 2, direction: $\mathbf{u} = 1/2\mathbf{i} + \sqrt{2}/2\mathbf{j} + 1/2\mathbf{k}$, $\mathbf{v} = 2\mathbf{u}$.

#14: velocity: $-e^{-t}\mathbf{i} - 6\sin(3t)\mathbf{j} + 6\cos(3t)\mathbf{k}$, acceleration: $e^{-t}\mathbf{i} - 18\cos(3t)\mathbf{j} - 18\sin(3t)\mathbf{k}$. At $t = 0$, speed = $\sqrt{37}$, direction: $\mathbf{u} = -1/\sqrt{37}\mathbf{i} + 6/\sqrt{37}\mathbf{k}$, $\mathbf{v} = \sqrt{37}\mathbf{u}$.

#16: 135^{circ}

#20: at all times $t \geq 0$.

#34: $x = 2t, y = 2, z = 20\pi + 5t$.

#40: Let $a = \sqrt{6}/3$. Then $\mathbf{r}(t) = (t^2 + (2/3)at + 1)\mathbf{i} + (t^2/2 + at - 1)\mathbf{j} + (t^2/2 + at + 2)\mathbf{k}$.

Section 13.2

#2: 490 m/s.

Section 13.3

#4: $\frac{1}{\sqrt{3}}(\mathbf{i} - \mathbf{j} + \mathbf{k}), 3\sqrt{3}$.

#12: $s = t^2$. arc length = $\frac{3}{4}\pi^2$.

#16: Cut along the line joining the points corresponding to $t = 0$ and $t = 2\pi$.

Section 14.1

#2: (a) domain: $y \geq x$. (b) all nonnegative real numbers. (c) lines with slope 1 and y -intercept ≥ 0 . (d) the line $y = x$. (e) closed region. (f) unbounded.

#6: (a) domain: $x \neq 0$ i.e. all points not on the y -axis. (b) all real numbers. (c) parabolas of the form $y = kx^2$, and the x -axis. (d) y -axis. (e) open region. (f) unbounded.

#14: e.

#16: c.

#18: b.

#30: $x = 1$.

#32: $y = 2x, x > 0$

Section 14.2

#6: 1.

#12: -2.

#14: 2.

#28: points not on the line $y = x$.

#52: 1.

Section 14.3

#10: $\frac{y^2-x^2}{(x^2+y^2)^2}, \frac{-2xy}{(x^2+y^2)^2}$.

#14: $e^{-x}(\cos(x+y) - \sin(x+y)), e^{-x} \cos(x+y)$.

#42: $f_{xx} = -y^2 \cos(xy), f_{xy} = f_{yx} = \cos(xy) - xy \sin(xy), f_{yy} = -x^2 \sin(xy)$.

Section 14.4

#4: By chain rule:

$\frac{dw}{dt} = \frac{2x}{x^2+y^2+z^2}(-\sin t) + \frac{2y}{x^2+y^2+z^2}(\cos t) + \frac{2z}{x^2+y^2+z^2} \frac{2}{\sqrt{t}}$. Substituting $x = \cos t, y = \sin t, z = 4\sqrt{t}$ and simplifying using $\sin^2 t + \cos^2 t = 1$, one gets $\frac{dw}{dt} = \frac{16}{1+16t}$.

Or substituting $x = \cos t, y = \sin t, z = 4\sqrt{t}$ into w and simplifying, one gets $w = \ln(1 + 16t)$, from which it follows $\frac{dw}{dt} = \frac{16}{1+16t}$.

(b) 16/49.

#8: By chain rule:

$$\begin{aligned} \frac{\partial z}{\partial u} &= \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u} \\ &= \frac{y}{x^2 + y^2} \cos v + \frac{-x}{x^2 + y^2} \sin v. \end{aligned}$$

Substituting $x = u \cos v, y = u \sin v$ and simplifying, one gets $\frac{\partial z}{\partial u} = 0$. Similarly, $\frac{\partial z}{\partial v} = -1$.

Or substituting $x = u \cos v, y = u \sin v$ into z and simplifying, one get $z = \tan^{-1}(\cot v) = \tan^{-1}(\tan(\pi/2 - v)) = \pi/2 - v$, from which it follows $\frac{\partial z}{\partial u} = 0$ and $\frac{\partial z}{\partial v} = -1$.

(b): 0, -1.

#10: $\frac{\partial w}{\partial u} = 2/u, \frac{\partial w}{\partial v} = 2$. (b) -1, 2.

#26: 2.

Section 14.5

#2: $\langle 1, 1 \rangle, x^2 + y^2 = 2$.

#4: $\langle \sqrt{2}, -1 \rangle, x^2 - y^2 = 1$.

#12: $-\frac{3}{2\sqrt{13}}$.

#14: 0

#18: $\mathbf{j}, -\mathbf{j}, 2, -2$

Section 14.6

#2: $3x + 5y + 4z = 18$, line: $x = 3 + 6t, y = 5 + 10t, z = -4 + 8t$. Note that the answer $x = 3 + 3t, y = 5 + 5t, z = -4 + 4t$ is also correct.

#4: $2y + 3z = 7$, line: $x = 1, y = -1 + 4t, z = 3 + 6t$. Note that the answer $x = 1, y = -1 + 2t, z = 3 + 3t$ is also correct.

#50: a: $dR = (R/R_1)^2 dR_1 + (R/R_2)^2 dR_2$. b: more sensitive to a variation in R_1 . c: about 0.1%.

Section 14.7

#10: local minimum at $(13/12, -3/4)$

#18: saddle point at $(0, 0)$, local maximum at $(-1, -1)$.

#34: absolute max. is 19 at $(5, 3)$; absolute min. is -12 at $(4, -2)$.

Section 15.1

#2: 0

#6: $\pi/4$.

#16: $1/4$

#32: $4 - \sin 4$

#38: $\frac{\ln 17}{4}$

#42: $63/20$

#44: $(9\pi - 8)/3$

Section 15.3

#2: π

#4: $\pi/2$

#6: 2π

#8: $4/3$

#18: $(8 + \pi)/4$

#20: $(64\pi^3)/27$

#22: $\frac{3\pi}{2} - 4$

Section 15.4

#24: $2/3$

#26: $2/3$

Section 16.1

#10: $-\sqrt{2}$

#12: 80π

#14: 1

Section 16.2

#18: 1

#20: $-\pi/2$

#26: $\frac{4}{3}a^3$

Section 16.3

#2: conservative
#4: not conservative
#8: $xy + xz + yz$
#10: $xy \sin z$
#14: -2

Section 16.4

#6: -3
#8: $-7/6$
#16: 16π
#18: -2
#22: πab