

**Final Exam Answers**

Math 351

July 28, 2005

Lim

1. If 5 cards are randomly selected from the standard 52-card deck, what is the probability that they are all of the same suit?

$5148/2598960$

2. How many ways can 4 boys and 6 girls be lined up, if the boys must be together?

$120960$

3. How many ways can 16 men be divided into three groups of respective size 3,6,7?

$960960$

4. How many integer solutions does the following inequality have

$$x_1 + x_2 + x_3 + x_4 \leq 45$$

if the solutions must also satisfy  $x_1 \geq 0, x_2 \geq 1, x_3 \geq 2, x_4 \geq 1$ . (Hint: Add another variable  $x_5$  to make  $x_1 + \dots + x_5 = 45$ .)

$148995$

5. How many ways can a group of 3 students be chosen from a class of 40 students to work on a project?

$9880$

6. How many 4-letter English words are possible if all letters must be different?

$358800$

7. How many 5-card poker hands are void in at least one suit?

$1913496$

8. An urn contains 4 white balls and 9 black balls. A ball is drawn at random. What is the probability that it is white?

$4/13$

9. An urn contains 4 white balls and 9 black balls. Three balls are drawn one by one without replacement. What is the probability that the third ball is white?

$4/13$

10. Two fair dice are rolled. Given that the sum of the scores is greater than or equal to 8, what is the probability at least one die lands on 4?

$\boxed{1/3}$

11. Suppose that 6 percent of men and 0.5 percent of women are colorblind. A colorblind person is chosen at random. What is the probability of this person being female? Assume that there are an equal number of males and females.

$\boxed{1/13}$

12. Suppose that each child born to a couple is equally likely to be a boy or a girl independent of the sex distribution of the other children in the family. For a couple having 4 children, what is the probability that all children are of the same sex?

$\boxed{1/8}$

13. The probability that A hits the target is  $1/5$  and the probability that B hits the target is  $1/3$ . If they both shoot at the target, what is the probability that at least one of them hit the target?

$\boxed{7/15}$

14. A sample of 3 items is selected at random from a box containing 10 items of which 4 are defective. Find the expected number of defective items in the sample.

$\boxed{1.2}$

15. If  $X$  is a random variable with  $E(X) = 2, Var(X) = 0.3$ , find the expected value of  $2X + 3$ .

$\boxed{7}$

16. (continued) Find the variance of  $2X + 3$ .

$\boxed{1.2}$

17. If you roll a die until you get a 6. What is the probability that you have to roll at least six times?

$\boxed{3125/7776}$

18. The probability of being dealt a full house in a hand of poker is approximately 0.0014. Use Poisson distribution to approximate the probability that in 1000 hands of poker you will be dealt at least 2 full houses.

$\boxed{0.4082}$

19. If you flip a fair coin 6 times, what is the probability that head appears exactly 3 times?
20. You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. What is the probability that you will have to wait longer than 15 minutes?
21. (continued) If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?
22. If  $X$  is a normal random variable with parameters  $\mu = 10$  and  $\sigma^2 = 36$ , compute  $P\{4 < X < 16\}$ .
23. If 65 percent of the population of a large community is in favor of a proposed rise in school taxes, approximate the probability that a random sample of 100 people will contain between 60 and 70 inclusive who are in favor.
24. If  $X$  is uniformly distributed over  $(-1, 1)$ , find the density function of the random variable  $Y = X^2$ . (All functions given below are 0 outside of the interval  $(0,1)$ .)
25. A television store owner figures that 38 percent of the customers entering his store will purchase an ordinary television set, 18 percent will purchase a plasma television set, and 44 percent will just be browsing. If 7 customers enter his store on a given day, what is the probability that he will sell exactly 2 ordinary sets and 1 plasma set on that day?
26. A man and a woman agree to meet at a certain location about 12:30 pm. If the man arrives at a time uniformly distributed between 12:15 and 12:45 and if the woman independently arrives at a time uniformly distributed between 12:00 and 1 pm, find the probability that the first

to arrive waits longer than 10 minutes.

$$\boxed{2/3}$$

27. The joint density function of  $X$  and  $Y$  is  $f(x, y) = x + y, 0 < x < 1, 0 < y < 1, 0$  elsewhere. Find the probability that  $X + Y < 1/2$ .

$$\boxed{1/24}$$

28. (continued) Find the conditional probability that  $X < 1/2$  given  $Y = 1/2$ .

$$\boxed{3/8}$$

29. A 5-card poker hand is called Two Pairs if it has 2 cards of one denomination, 2 cards of another denomination, and 1 card of a third denomination. For example, two 5's , two kings, and one ace. Find the probability of a Two Pairs.

$$\boxed{123552/2598960}$$