Math 213 Exam 2 Answers

1. An equation in spherical coordinates of the sphere of radius 2 centered at $(0,0,2)$ is
$\boxed{\rho = 4\cos\phi}$

2. Let D be the region bounded by the sphere $x^2+y^2+z^2=1$ and the x-y plane in the upper half space (see fig. 3). The triple integral $\int\int\int_D z\,dV$ is equal to

$\boxed{\int_0^{2\pi}\int_0^{\pi/2}\int_0^1 \rho^3\cos\phi\sin\phi\,d\rho\,d\phi\,d\theta}$

3. Find the critical point of the function $f(x,y)=x^2+5xy-10x$.
$\boxed{(0,2)}$

4. Compute $\int_C x+y\,ds$, where C is the curve given by: $x=t, y=t+t^2, 0\le t\le 1$. The answer is given by the integral:
$\boxed{\int_0^1 t(t+2)\sqrt{4t^2+4t+2}\,dt}$

5. Minimize x^2+y^2 subject to $x^2y=1$. To solve this problem by Lagrange Multiplier Method, we need to solve the following simultaneous equations for x,y:
$\boxed{2x+2\lambda xy=0, 2y+\lambda x^2=0, x^2y-1=0}$

6. Evaluate $\int_0^2\int_{2x}^4 x+2y\,dy\,dx$.
$\boxed{24}$

7. Let R be the region bounded by the circle $x^2+y^2=1$ and the x-axis, in the upper half plane (see fig. 2). Evaluate $\int\int_R\sqrt{x^2+y^2}\,dA$.
$\boxed{\frac{1}{3}\pi}$

8. Let R be the region bounded by the circle $x^2+y^2=1$ and the x-axis, in the upper half plane (see fig. 2). The integral $\int\int_R x^2y\,dA$ is equal to
$\boxed{\int_0^{\pi}\int_0^1 r^4\cos^2\theta\sin\theta\,dr\,d\theta}$

9. Reverse the order of integration $\int_{-1}^1\int_{1-x}^2 f(x,y)\,dy\,dx$.

$\boxed{\int_0^2\int_{1-y}^1 f(x,y)\,dx\,dy}$

10. The volume of the solid bounded by the surface $z = 9 - x^2 - y^2$ and the x-y plane (see fig. 5) is given by the integral:
$\boxed{\int_0^{2\pi} \int_0^3 (9 - r^2) r \, dr \, d\theta}$

11. It is known that $P = (2, 1)$ is a critical point of $f(x, y) = x^2 - 4xy + y^2 + 6y + 2$. Which of the following is true?
$\boxed{P \text{ is a saddle point of } f.}$

12. Let D be the region in the first octant bounded by the coordinate planes and the planes $x + z = 1, y + 2z = 2$ (see fig. 4). Then the integral $\int\int\int_D f(x, y, z) \, dV$ is equal to
$\boxed{\int_0^1 \int_0^{1-x} \int_0^{2-2z} f(x, y, z) \, dy \, dz \, dx}$

13. Find the gradient vector of $f(x, y) = xy^2 + 3y^3$ at $(2, -1)$.

$\boxed{\langle 1, 5 \rangle}$

14. Find the rate of change of the function $f(x, y, z) = xyz$ at $(1, -1, 2)$ in the direction of the vector $\langle 1, 2, 2 \rangle$, i.e. in the direction $\langle 1/3, 2/3, 2/3 \rangle$.
$\boxed{0}$

15. The least (most negative) directional derivative of the function $f(x, y) = xy + y^2$ at $(3, 2)$ is
$\boxed{-\sqrt{53}}$

16. Let $f(x, y) = x^2 + 2y^2$. An equation of the tangent plane to the graph of the function $z = f(x, y)$ at the point $(1, 2, 9)$ is
$\boxed{z = 9 + 2(x - 1) + 8(y - 2)}$

17. Find a vector that is perpendicular to the surface $x^2y + y^2z = 0$ at the point $(1, -1, 1)$.
$\boxed{\langle -2, -1, 1 \rangle}$

18. If $z = \ln(xy^2)$, find dz.
$\boxed{\frac{1}{x} dx + \frac{2}{y} dy}$

19. Evaluate $\int\int_R 2xy \, dA$, where R is the rectangle: $0 \leq x \leq 2, 0 \leq y \leq 1$.
$\boxed{2}$

20. Suppose R is the region bounded by $y = x^3, x = 2$ and $y = 0$ (see fig. 1). Then the integral $\int\int_R f(x,y)\,dA$ is equal to

$\int_0^2 \int_0^{x^3} f(x,y)\,dy\,dx$

21. Suppose R is the region bounded by $y = x^3, x = 2$ and $y = 0$ (see fig. 1). Then the integral $\int\int_R f(x,y)\,dA$ is equal to

$\int_0^8 \int_{\sqrt[3]{y}}^2 f(x,y)\,dx\,dy$