

Math 213 Exam 1

Note: Write your name on both the exam paper and your answer sheet because this exam has different versions. NA means none of the above.

1. Compute the \mathbf{j} -component of $\langle 1, 2, 3 \rangle \times \langle -2, 3, 4 \rangle$.
2. True or False: $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.
3. True or False: If the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ exist at (a, b) , then f is continuous at (a, b) .
4. Find a tangent vector to the curve: $\mathbf{r}(t) = t^2\mathbf{i} + \cos(2t)\mathbf{j} + 3t\mathbf{k}$ at $t = 1$.
5. Let $\mathbf{u} = \langle 2, 1, 4 \rangle$ and $\mathbf{v} = \langle 3, -2, 4 \rangle$. Find $\mathbf{u} \cdot \mathbf{v}$.
6. The line $4x + 3y = 9$ in the x - y plane is
7. Find an equation of the plane through $(1, 2, 3)$ that is perpendicular to the line: $x = 1 + 2t, y = 3 + t, z = 2 + 4t$.
8. Derive a formula for the distance from (a, b, c) to the plane $Ax + By + Cz = D$.
9. The area of the parallelogram formed by two vectors \mathbf{u}, \mathbf{v} is
10. Let $\mathbf{u} = \langle 2, -2, 3 \rangle$. Compute $|\mathbf{u}|^2$.
11. Let $f(x, y) = x^2y + 2xy^3 + 2y + x$. Find $\frac{\partial f}{\partial y}$.
12. Let $f(x, y) = x^3y^2 + 2xy + 5y$. Find $\frac{\partial f}{\partial x}(2, 1)$.

13. In the Fig. 1, the vector \mathbf{w} is equal to

$$\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2} \mathbf{v}$$

14. An object with mass m is moving under a constant force $\mathbf{F} = m(4\mathbf{i} - 32\mathbf{j})$. Assume that its initial position is $\mathbf{0}$ and its initial velocity is $2\mathbf{i} + 4\mathbf{j}$. Find its position vector $\mathbf{r}(t)$.

$$2t(t+1)\mathbf{i} + 4t(-4t+1)\mathbf{j}$$

15. Find the arc length of the space curve given by the parametric equations: $x = t, y = t^2, z = t^3, 0 \leq t \leq 1$.

$$\int_0^1 \sqrt{1 + 4t^2 + 9t^4} dt$$

16. The angle between the two vectors $\langle 1, 2, 3 \rangle$ and $\langle 1.5, 0.7, -1 \rangle$ is

obtuse

17. Find the (acute) angle between the two planes: $x + 2y + 3z = 1$ and $2x + y - 3z = 4$.

$$\cos^{-1}(5/14)$$

18. Find the volume of the parallelepiped determined by the three vectors: $\langle 1, 2, 3 \rangle$, $\langle -2, 1, -3 \rangle$, and $\langle 1, 1, 1 \rangle$.

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19. Let $f(x, y), g(t), h(t)$ be differentiable functions. Let $m(t) = f(g(t), h(t))$. The Chain Rule says

$$m'(t) = \frac{\partial f}{\partial x}(g(t), h(t))g'(t) + \frac{\partial f}{\partial y}(g(t), h(t))h'(t)$$

20. (continued) Suppose $g(0) = 2, h(0) = 7, \frac{\partial f}{\partial x}(2, 7) = 3, \frac{\partial f}{\partial y}(2, 7) = -4, \frac{\partial f}{\partial x}(0, 0) = 1, \frac{\partial f}{\partial y}(0, 0) = -5, g'(0) = 1, h'(0) = -6, f(0, 0) = 9, f(2, 7) = 11$, find $m'(0)$.

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