Section 1. **Multiple Choice Problems** (each problem is 1 point)

1. How many four-letter words are possible if adjacent letters must be different? e.g. FOUR, FOUF, BOBO are allowed, but AOOF, CCOU, DDDU are not.
   \[26 \cdot 25 \cdot 25 \cdot 25 = 406250\]

2. How many three-digit numbers are possible if the first digit cannot be 0? e.g. 100, 303, 321, 333 are allowed, but 012, 000, 011 are not.
   \[900\]

3. How many ways can 4 students be selected from a class of 20 students to clean the floor?
   \[4845\]

4. Roses come in 10 different colors. How many different bunches of 5 roses can be formed?
   \[2002\]

5. From a class of 36 students, 6 are to be selected to clean the floor, 3 to clean the board, and 7 to paint the wall. How many ways can this be done?
   \[\frac{36!}{6!3!7!20!}\]

6. There are five sets \(A, B, C, D, E\) each with 20 elements; intersection of any two of them has 7 elements; intersection of any three of them has 4 elements; intersection of any four of them has 3 elements, and \(A \cap B \cap C \cap D \cap E\) has 2 element. How many elements does \(A \cup B \cup C \cup D \cup E\) have?
   \[57\]

7. There are 251 pigeons in 48 holes. By the strong form of the Pigeon Hole Principle one of the holes must have at least \[\text{........} \text{ pigeons}\]
   \[6\]

8. Let \(A = \{1, 2, 3, 4, 5, 6, 7\}\). Define a relation \(\equiv\) on \(A\) as follows: \(a \equiv b\) if and only if \(a - b\) is divisible by 3. (A number \(n\) is divisible by 3 if and only if \(n = 3m\) for some integer \(m\). e.g. \(-9, 15\) are divisible by 3.) The equivalence classes induced by \(\equiv\) on \(A\) are:
   \[\{1, 4, 7\}, \{2, 5\}, \{3, 6\}\]
9. Let \( A = \{1, 2, 3, 4\} \). Let \( R = \{(1, 1), (2, 2), (3, 3), (3, 4), (4, 3), (1, 3), (3, 1)\}\). Which of the following properties does \( R \) have? (a) reflexive, (b) symmetric, (c) transitive, (d) antisymmetric.

(b) only

10. Find the number of permutations of the digits 1, 2, \cdots, 9 in which at most one digit is in its proper position.

266993

11. There are 6 different roads from city A to city B and 7 different roads from city B to city C. How many different round trips are there from City A to City C and back, passing through City B each way, and you don’t want to drive on any road more than once?

1260

12. The sum of the first two terms of the expansion of \((x + y)^9\) is \(x^9 + 9x^8y\). What is the third term?

\[36x^7y^2\]

13. A connected pseudograph has an Eulerian circuit if and only if the degree of each vertex is

even

14. True or False: A connected graph with 8 vertices has a Hamiltonian cycle if the degree of each vertex is greater than or equal to 4.

True

15. True or False: If a connected graph has 6 vertices and one of the vertex has degree 2, then no Hamiltonian cycles exist.

False

16. A tree is a connected graph which contains

no circuits

17. A subgraph of a connected graph \( G \) is called a spanning tree if it is a tree containing

all vertices of \( G \)

18. In applying Dijkstra’s algorithm for finding a shortest path from A to E in Fig. 1, the first two labeled vertices are \( A(−, 0) \) and \( G(A, 2) \). The next vertex to be labeled should be

\( I(G, 4) \)
19. In applying Kruskal’s algorithm for finding a minimum spanning tree in Fig. 2, the first 6 selected edges are AI, BC, CK, BJ, KL and AB. Which should be the next edge to be selected? DE or EF

Section 2. Other Problems (each problem is 2 point)

1. Find an Eulerian trail in Fig. 3 if possible.
   FCECDCBDEDFDABBA (other answers are possible)

2. Find a Hamilton cycle in Fig. 4 if possible.
   BAFEDIHGCB

3. Prove by induction that $4^n - 1$ is divisible by 3 for all positive integers $n$.
   For $n = 1$, $4^1 - 1 = 3$ is divisible by 3.
   Suppose it is true for $n = k$ for some $k$. Then $4^{k+1} - 1 = 4 \cdot 4^k - 1 = (3 + 1) \cdot 4^k - 1 = 3 \cdot 4^k + 4^k - 1$, and this is divisible by 3 because $3 \cdot 4^k$ is divisible by 3 and $4^k - 1$ is divisible by 3 by induction hypothesis. By induction, this proves that $4^n - 1$ is divisible by 3 for all positive integers $n$. 

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