Math 114 Exam 2 Review Problems

1. Evaluate the integral
   \[ \int_0^1 \frac{2x + 1}{x^2 + 4x + 8} \, dx \]
   Ans. \( \ln(x^2 + 4x + 8) - \frac{3}{2} \tan^{-1} \frac{x + 2}{2} \)

2. Evaluate the integral
   \[ \int_0^1 \frac{1}{(3x + 1)(2x + 1)} \, dx \]
   Ans. \( \ln \frac{1}{3} \)

3. Evaluate the integral
   \[ \int_1^2 \frac{1}{x \sqrt{x^2 - 1}} \, dx \]
   Ans. \( \pi/3 \)

4. Evaluate the integral
   \[ \int_1^2 \frac{1}{x^2 - 1} \, dx \]
   Ans. \( \infty \)

5. Evaluate the integral
   \[ \int_2^\infty \frac{3}{2x^2 + x - 1} \, dx \]
   Ans. \( \ln 2 \)

6. Evaluate the integral
   \[ \int_0^{\pi/2} \tan x \, dx \]
   Ans. \( \infty \)

7. Find the sum of the series, if it exists,
   \[ \sum_{n=1}^\infty \frac{2}{4n^2 - 1} \]
   Ans. 1
8. Find the sum of the series, if it exists,
\[ \sum_{n=1}^{\infty} \frac{2}{3^{n+1}} \]
Ans. 1/3

9. Define the sequence recursively by: \( a_0 = \sqrt{2}, a_{n+1} = \sqrt{2 + a_n} \) for \( n = 1, 2, \cdots \). Prove that the sequence converges and find its limit.
Ans. 2

10. Determine whether the following series is convergent:
\[ \sum_{n=1}^{\infty} \frac{\ln n}{n} \]
Ans. diverge

11. Determine whether the following series is convergent:
\[ \sum_{n=1}^{\infty} \frac{n}{n + 1} \]
Ans. diverge

12. Determine whether the following series is convergent:
\[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \]
Ans. diverge

13. Determine whether the following series is convergent:
\[ \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \]
Ans. converge

14. If \( S_{100} = \sum_{n=2}^{100} \frac{1}{n(\ln n)^2} \), the 100th partial sum of the series
\[ \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \]
is used as an approximation of the sum \( S \) of the series, find an upper and a lower bound for the error \( S - S_{100} \).
Ans. lower bound 1/(ln 101), upper bound 1/(ln 100).