11.9 314 hw9

#2 $\mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{kx} e^{-i\omega x}\,dx = \frac{e^{(k-i\omega)x}}{\sqrt{2\pi}(k-i\omega)}\Big|_{-\infty}^{0} = \frac{1}{\sqrt{2\pi}(k-i\omega)}$

#4 $\mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{2ix} e^{-i\omega x}\,dx = \frac{1}{\sqrt{2\pi}} \frac{e^{i(2-\omega)x}}{i(2-\omega)}\Big|_{-1}^{1} = \frac{1}{\sqrt{2\pi}} \frac{e^{i(2-\omega)} - e^{-i(2-\omega)}}{i(2-\omega)}$

$= \frac{\cos(2-\omega) + i\sin(2-\omega) - \cos(2-\omega) + i\sin(2-\omega)}{\sqrt{2\pi}\; i(2-\omega)} = \sqrt{\frac{2}{\pi}} \sin(2-\omega)/(2-\omega)$

#6 $\mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} x e^{-i\omega x}\,dx = \frac{1}{\sqrt{2\pi}} \left[\frac{x e^{-i\omega x}}{-i\omega} - \frac{e^{-i\omega x}}{-\omega^2} \right]_{-1}^{1}$

$= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{-i\omega}}{-i\omega} + \frac{e^{i\omega}}{-i\omega} + \frac{e^{-i\omega}}{\omega^2} - \frac{e^{i\omega}}{\omega^2} \right] = \frac{1}{\sqrt{2\pi}} \left[\frac{2\cos\omega}{-i\omega} - \frac{2i\sin\omega}{\omega^2} \right]$

$= \sqrt{\frac{2}{\pi}} \frac{i}{\omega^2} [\omega\cos\omega - \sin\omega]$

#10 Let $g(x) = e^{-x}$ if $x > 0$ and 0 if $x < 0$. Then $f' = g - f$

So $i\omega\,\mathcal{F}(f) = \mathcal{F}(f') = \mathcal{F}(g) - \mathcal{F}(f)$ So $(i\omega + 1)\,\mathcal{F}(f) = \mathcal{F}(g) = \frac{1}{\sqrt{2\pi}(1+i\omega)}$

Hence $\mathcal{F}(f) = \frac{1}{\sqrt{2\pi}(1+i\omega)^2}$ Compare with #8.

12.1 #2: Sol of $u'' = u$ is $u = c_1 e^{x} + c_2 e^{-x}$ So $u = c_1(y)e^{x} + c_2(y)e^{-x}$

#6: Sol of $u'' = k^2 u$ is $u = c_1 e^{kx} + c_2 e^{-kx}$. So $u = c_1(y)e^{2yx} + c_2(y)e^{-2yx}$

#10: Sol of $u'' = ku'$ is $u = c_1 + c_2 e^{ky}$. So $u = c_1(x) + c_2(x)e^{4xy}$

#14: $u_{tt} = 2$; $u_{xx} = 8$. $\therefore$ $u_{tt} = \frac{1}{4}u_{xx} = c^2 u_{xx}$ with $c = \frac{1}{2}$

#18: $u_x = -2k e^{-kt}\cos 8t$; $u_{xx} = -64 e^{-kt}\cos 8t$

So $u_t = \frac{k}{32} u_{xx} = c^2 u_{xx}$ with $c = \sqrt{k/32}$

#24 $u_x = \frac{1}{1+(\frac{y}{x})^2}\left(\frac{-y}{x^2}\right) = \frac{-y}{x^2+y^2}$; $u_{xx} = \frac{2xy}{(x^2+y^2)^2}$

$u_y = \frac{1}{1+(\frac{y}{x})^2}\left(\frac{1}{x}\right) = \frac{x}{x^2+y^2}$; $u_{yy} = \frac{-2xy}{(x^2+y^2)^2}$

$\therefore u_{xx} + u_{yy} = 0$

TAKE NOTE: $\arctan\frac{y}{x}$ is multi-valued, so not really a function. $\text{Arctan}\frac{y}{x}$, or any other single valued branch of $\arctan\frac{y}{x}$, is a solution.