11.7 314 hw 8

#2 $B(w) = \frac{2}{\pi}\int_0^1 \pi \frac{x}{2} \sin wx \, dx = \left[-x \frac{\cos wx}{w} + \frac{\sin wx}{w^2}\right]_0^1$

$= \frac{\sin w}{w^2} - \frac{\cos w}{w}$ and $f(x) = \int_0^{+\infty} B(w) \sin wx \, dw$

#4 $A(w) = \frac{2}{\pi}\int_0^1 \frac{\pi}{2} \cos wx \, dx = \frac{\sin w}{w}$ and $f(x) = \int_0^{+\infty} A(w) \cos wx \, dw$

#8 $A(w) = \frac{2}{\pi}\int_0^a x^2 \cos wx \, dx = \frac{2}{\pi}\left[x^2 \frac{\sin wx}{w} + \frac{2x \cos wx}{w^2} - \frac{2 \sin wx}{w^3}\right]_0^a$

$= \frac{2}{\pi}\left[a^2 \frac{\sin wa}{w} + \frac{2a \cos wa}{w^2} - \frac{2 \sin wa}{w^3}\right]$ and $f(x) = \int_0^{+\infty} A(w) \cos wx \, dw$

#12 $A(w) = \frac{2}{\pi}\int_0^a e^{-x} \cos wx \, dx = \frac{2e^{-x}}{\pi(1+w^2)}\left[-\cos wx + w \sin wx\right]_0^a$

$= \frac{2}{\pi(1+w^2)}\left[e^{-a}(w \sin wa - \cos wa) + 1\right]$ and $f(x) = \int_0^{+\infty} A(w) \cos wx \, dw$

#14 $B(w) = \frac{2}{\pi}\int_0^a \sin wx \, dx = \frac{-2}{\pi w} \cos wx \Big|_0^a = \frac{2}{w\pi}(1 - \cos wa)$

and $f(x) = \int_0^{+\infty} B(w) \sin wx \, dx$

11.8

#2 $\sqrt{\frac{2}{\pi}} \int_0^k x \cos wx \, dx = \sqrt{\frac{2}{\pi}}\left[x \frac{\sin wx}{w} + \frac{\cos wx}{w^2}\right]_0^k = \sqrt{\frac{2}{\pi}}\left[\frac{k \sin wk}{w} + \frac{\cos wk}{w^2} - \frac{1}{w^2}\right.$

#4 Let $g(w) = \sqrt{\frac{2}{\pi}}\left[\frac{\sin 2w - 2 \sin w}{w}\right]$ from problem 1

Then $\mathcal{F}_c(g) = \frac{2}{\pi}\int_0^{+\infty} \frac{(\sin 2w - 2\sin w)}{w} \cos wx \, dw =$

$\frac{2}{\pi}\left[\int_0^{+\infty} \frac{\sin v \cos(vx/2)}{v} dv - 2\int_0^{+\infty} \frac{\sin w \cos wx}{w} dw\right]$. By 4 of 11.7

first integral is $\begin{cases} \frac{\pi}{2} & \text{if } \frac{x}{2} < 1 \\ 0 & \text{if } \frac{x}{2} > 1 \end{cases}$ second integral is $\begin{cases} \frac{\pi}{2} & \text{if } x < 1 \\ 0 & \text{if } x > 1 \end{cases}$

For $0 < x < 1$, $\frac{2}{\pi}\left[\frac{\pi}{2} - 2\frac{\pi}{2}\right] = -1$; For $1 < x < 2$, $\frac{2}{\pi}\left(\frac{\pi}{2} - 0\right) = 1$;

For $2 < x$, $\frac{2}{\pi}(0 - 0) = 0$. Note these agree with defin of $f(x)$

#8 $\mathcal{F}_c(f) = \sqrt{\frac{\pi}{2}} A(w)$ where $A(w)$ is answer to 11.7#8 with $a = 1$.

#12 $\pi^2 \mathcal{F}_s(e^{-\pi x}) = \mathcal{F}_s((e^{-\pi x})'') = -w^2 \mathcal{F}_s(e^{-\pi x}) + \sqrt{\frac{2}{\pi}} w$; $\mathcal{F}_s(e^{-\pi x}) = \sqrt{\frac{2}{\pi}} \frac{w}{\pi^2 + w^2}$

#14 $\sqrt{\frac{2}{\pi}}\int_0^\pi \sin x \sin wx \, dx = \sqrt{\frac{2}{\pi}} \frac{1}{2}\int_0^\pi [\cos(w-1)x - \cos(w+1)x] \, dx =$

$\sqrt{\frac{2}{\pi}}\frac{1}{2}\left[\frac{\sin(w-1)\pi}{w-1} - \frac{\sin(w+1)\pi}{w+1}\right] = \sqrt{\frac{2}{\pi}}\frac{1}{2} \sin w\pi\left(\frac{1}{w+1} - \frac{1}{w-1}\right) = \sqrt{\frac{2}{\pi}} \sin w\pi\left(\frac{-1}{w^2-1}\right)$

So $\int_0^{+\infty} (\sin w\pi \sin wx)/(1-w^2) \, dw = \frac{\pi}{2} f(x)$ as given in #6 of 11.7