11.2

#4 $b_n = \int_{-1}^{1} \frac{\pi}{2} x^3 \sin \pi n x \, dx = -\frac{1}{2n} x^3 \cos n\pi x \Big|_{-1}^{1} + \frac{1}{2n}\int_{-1}^{1} 3x^2 \cos n\pi x \, dx$

$= (-1)^{n+1}\frac{1}{n} + \frac{3}{2n^2\pi} \sin n\pi x \Big|_{-1}^{1} - \frac{3}{2n^2\pi}\int_{-1}^{1} 2x \sin n\pi x \, dx =$

$(-1)^{n+1}\frac{1}{n} + \frac{3}{n^3\pi^2} x \cos n\pi x \Big|_{-1}^{1} - \frac{3}{n^3\pi^2}\int_{-1}^{1} \cos n\pi x \, dx =$

$(-1)^{n+1}\frac{1}{n} + (-1)^n \frac{6}{n^3\pi^2}$; $a_n = 0$

So $\sum_{n=1}^{+\infty}\left[(-1)^{n+1}\frac{1}{n} + (-1)^n\frac{6}{n^3\pi^2}\right]\sin n\pi x = \left(1 - \frac{6}{\pi^2}\right)\sin \pi x + \left(-\frac{1}{2} + \frac{6}{2^3\pi^2}\right)\sin 2\pi x$

$+ \left(\frac{1}{3} - \frac{6}{3^3\pi^2}\right)\sin 3\pi x + \cdots$

#8 $a_0 = \frac{1}{2} 2\int_0^1 (1-x)\,dx = x - \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$; $b_n = 0$

$a_n = 2\int_0^1 (1-x)\cos n\pi x \, dx = 2\left[\frac{\sin n\pi x}{n\pi}\Big|_0^1 - \frac{x \sin n\pi x}{n\pi}\Big|_0^1 + \int_0^1 \frac{\sin n\pi x}{n\pi}\,dx\right] =$

$= -2\left(\frac{\cos n\pi x}{n^2\pi^2}\right)\Big|_0^1 = \frac{2}{n^2\pi^2}\left[1 - (-1)^n\right] = \begin{cases} 4/(n^2\pi^2) & \text{for } n \text{ odd} \\ 0 & \text{for } n \text{ even}\end{cases}$

$\frac{1}{2} + \sum_{k=0}^{+\infty}\frac{4}{\pi^2}\frac{\cos(2k+1)\pi x}{(2k+1)^2} = \frac{1}{2} + \frac{4}{\pi^2}\left[\frac{\cos \pi x}{1} + \frac{\cos 3\pi x}{9} + \frac{\cos 5\pi x}{25} + \cdots\right]$

#10 $a_0 = \frac{1}{4}\int_0^2 x\,dx = \frac{1}{4}\frac{x^2}{2}\Big|_0^2 = \frac{1}{2}$

$a_n = \frac{1}{2}\int_0^2 x \cos\frac{n\pi x}{2}\,dx = \frac{1}{n\pi} x \sin\frac{n\pi x}{2}\Big|_0^2 - \frac{1}{n\pi}\int_0^2 \sin\frac{n\pi x}{2}\,dx$

$= \frac{2}{n^2\pi^2}\cos\frac{n\pi x}{2}\Big|_0^2 = \frac{2}{n^2\pi^2}\left[-1 + (-1)^n\right]$

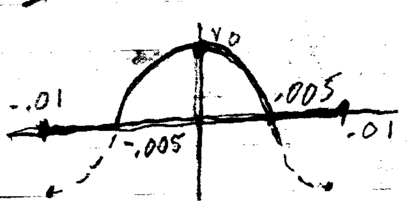
$b_n = \frac{1}{2}\int_0^2 x \sin\frac{n\pi x}{2}\,dx = \frac{-1}{n\pi} x \cos\frac{n\pi x}{2}\Big|_0^2 + \frac{1}{n\pi}\int_0^2 \cos\frac{n\pi x}{2}\,dx$

$= -\frac{2}{n\pi}(-1)^n + \frac{2}{n^2\pi^2}\sin\frac{n\pi x}{2}\Big|_0^2 = \frac{2}{n\pi}(-1)^{n+1}$

So $\frac{1}{2} - \frac{4}{\pi^2}\sum_{k=0}^{+\infty}\frac{\cos[(2k+1)\pi x/2]}{(2k+1)^2} + \frac{2}{\pi}\sum_{n=1}^{+\infty}(-1)^{n+1}\frac{\sin[n\pi x/2]}{n}$

$= \frac{1}{2} - \frac{4}{\pi^2}\left[\cos\pi x + \frac{\cos 3\pi x}{9} + \frac{\cos 5\pi x}{25} + \cdots\right] + \frac{2}{\pi}\left[\sin\pi x - \frac{\sin 2\pi x}{2} + \frac{\sin 3\pi x}{3} - \cdots\right]$

#12



$a_0 = 50\int_{-.005}^{.005} V_0 \cos 100\pi t\,dt = V_0/\pi$

$b_n = 100\int_{-.005}^{.005} V_0 \cos 100\pi t \sin 100\pi n t\,dt = 0$

$a_n = 100\int_{-.005}^{.005} V_0 \cos 100\pi t \cos 100\pi n t\,dt = 50V_0\int_{-.005}^{.005}\left[\cos 100\pi(n-1)t + \cos 100\pi(n+1)t\right]$

$= \frac{50V_0 2}{100\pi}\left[\frac{\sin .5\pi(n+1)}{n+1} + \frac{\sin .5\pi(n-1)}{(n-1)}\right]$ if $n \neq 1$ and $= V_0/2$ if $n = 1$ (over)

11.2 #12 cont $\frac{V_0}{\pi} + \frac{V_0}{2}\cos 100\pi t + \sum_{k=1}^{+\infty} \frac{(-1)^{k+1} 2V_0}{\pi(2k+1)(2k-1)} \cos 2k100\pi t =$

$\frac{V_0}{\pi} + \frac{V_0}{2}\cos 100\pi t + \frac{2V_0}{\pi}\left(\frac{1}{3\cdot 1}\cos 200\pi t - \frac{1}{5\cdot 3}\cos 400\pi t + \frac{1}{7\cdot 5}\cos 600\pi t + \cdots\right)$

11.3

#2 even, even, even, even, neither, neither, odd, odd

#4 $f\left(\frac{-3\pi}{4}\right) = f\left(\frac{5\pi}{4}\right) = \left(\frac{5\pi}{4}\right)^2 \neq \pm\left(\frac{3\pi}{4}\right)^2$, so neither

#6 $f(-x) = (-x)^3 \sin(-x) = f(x)$, so even

#12 $f(-x) = 2(-x)|-x| = -f(x)$, so odd

$b_n = 2\int_0^1 2x^2 \sin n\pi x\,dx = 4\frac{1}{n\pi}\left(-x^2\cos n\pi x\Big|_0^1 + \int_0^1 2x\cos n\pi x\,dx\right)$

$= \frac{4}{n\pi}\left(-\cos n\pi + \frac{2}{n\pi}x\sin n\pi x\Big|_0^1 - \frac{2}{n\pi}\int_0^1 \sin n\pi x\,dx\right) =$

$\frac{4}{n\pi}\left((-1)^{n+1} + \frac{2}{n^2\pi^2}\cos n\pi x\Big|_0^1\right) = \frac{4}{n\pi}\left((-1)^{n+1} + \frac{2}{n^2\pi^2}\left((-1)^n - 1\right)\right)$

So $\sum_{n=1}^{+\infty} \frac{4}{n\pi}(-1)^{n+1}\sin n\pi x + \sum_{k=0}^{+\infty} \frac{-16}{(2k+1)^3\pi^3}\sin(2k+1)\pi x =$

$\sin(\pi x)\left(\frac{4}{\pi} - \frac{16}{\pi^3}\right) - \sin(2\pi x)\left(\frac{2}{\pi}\right) + \sin(3\pi x)\left(\frac{4}{3\pi} - \frac{16}{3^3\pi^3}\right) - \sin(4\pi x)\left(\frac{1}{\pi}\right) + \cdots$

#14 $f(-x) = f(x)$ so even

$a_0 = \frac{2}{2\pi}\int_0^\pi \pi e^x\,dx = (e^\pi - 1)$; $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

$a_n = \frac{2}{\pi}\int_0^\pi \pi e^x \cos nx\,dx = \frac{2e^x}{1+n^2}(\cos nx + n\sin nx)\Big|_0^\pi$

$= \frac{2e^\pi}{(1+n^2)}(-1)^n - \frac{2}{1+n^2}$ So $(e^\pi - 1) + \sum_{n=1}^{+\infty} \frac{2}{1+n^2}\left(e^\pi(-1)^n - 1\right)\cos nx$

$= (e^\pi - 1) + (-e^\pi - 1)\cos x + \frac{2}{5}(e^\pi - 1)\cos 2x + \frac{1}{5}(-e^\pi - 1)\cos 3x + \cdots$

#18 a) $a_0 = 2\int_0^{\frac{1}{2}} x\,dx = \frac{1}{4}$

$a_n = 4\int_0^{\frac{1}{2}} x\cos 2n\pi x\,dx = \frac{4}{2n\pi}\left[x\sin 2n\pi x\Big|_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \sin 2n\pi x\,dx\right]$

$= \frac{2}{n\pi}\left(\frac{1}{2n\pi}\cos 2n\pi x\Big|_0^{\frac{1}{2}}\right) = \frac{1}{n^2\pi^2}\left[(-1)^n - 1\right]$ So $\frac{1}{4} - \frac{2}{\pi^2}\sum_{k=0}^{+\infty} \frac{\cos 2(2k+1)\pi x}{(2k+1)^2}$

$= \frac{1}{4} - \frac{2}{\pi^2}\left[\cos 2\pi x + \frac{1}{9}\cos 6\pi x + \frac{1}{25}\cos 10\pi x + \cdots\right)$

b) $b_n = 4\int_0^{1/2} x\sin 2n\pi x\,dx = \frac{2}{n\pi}\left[-x\cos 2n\pi x\Big|_0^{1/2} + \int_0^{\frac{1}{2}} \cos 2n\pi x\,dx\right]$

$= \frac{1}{n\pi}(-1)^{n+1}$ So $\sum_{n=1}^{+\infty} \frac{1}{n\pi}(-1)^{n+1}\sin 2n\pi x = \frac{1}{\pi}\left(\sin 2\pi x - \frac{1}{2}\sin 4\pi x + \frac{1}{3}\sin 6\pi x + \cdots\right)$

11.4 #9 $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} x e^{-inx}\,dx = \frac{-1}{2\pi i n}\left(x e^{-inx}\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} e^{-inx}\,dx\right)$

$= \frac{-1}{2\pi i n}\left(\pi e^{-in\pi} + \pi e^{in\pi} + \frac{1}{in} e^{-inx}\Big|_{-\pi}^{\pi}\right)$

$= \frac{-1}{2\pi i n}\left(2\pi\cos(n\pi)\right) = \frac{i}{n}(-1)^n$ for $n \neq 0$

$c_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\,dx = 0$ So $\sum_{n\neq 0} \frac{i}{n}(-1)^n e^{inx}$

#10 $\sum_{n=1}^{\infty}\left(\frac{i}{n}(-1)^n\left[\cos nx + i\sin nx\right] + \frac{i}{-n}(-1)^{-n}\left[\cos(-nx) + i\sin(-nx)\right]\right)$

$= \sum_{n=1}^{+\infty} (-1)^{n+1}\frac{2\sin nx}{n}$