5.8 314 hw 5

#4: $1=P_0(x)$; $x=P_1(x)$; $x^2=a_0+a_2(3x^2-1)/2$ ∴ $1=3a_2/2$; $0=a_0-a_2/2$

so $x^2=\frac{2}{3}P_2(x)+\frac{1}{3}P_0(x)$; $x^3=a_1x+a_3(5x^2-3x)/2$ ∴ $1=5a_3/2$

$0=a_1-3a_3/2$ so $x^3=\frac{2}{5}P_3(x)+\frac{3}{5}P_1(x)$

#2: $(x+1)^2=1+2x+x^2=P_0(x)+2P_1(x)+\frac{2}{3}P_2(x)+\frac{1}{3}P_0(x)=\frac{4}{3}P_0(x)+2P_1(x)+\frac{2}{3}P_2(x)$

#6: $a_{2k}=0$ since $P_{2k}(x)\sin\pi x$ is an odd fctn so $\int_{-1}^{1}P_{2k}(x)\sin\pi x=0$

$a_1=\frac{3}{2}\int_{-1}^{1}x\sin\pi x\,dx=\frac{3}{2}\left[(1+1)/\pi+\frac{1}{\pi}\int_{-1}^{1}\cos\pi x\,dx\right]=\frac{3}{\pi}\approx .955$

$a_3=\frac{7}{2}\cdot\frac{1}{2}\int_{-1}^{1}(5x^3-3x)\sin\pi x\,dx=\frac{7}{4}\left[\frac{10}{\pi}-\frac{6}{\pi}+\frac{1}{\pi}\int_{-1}^{1}\cos\pi x(15x^2-3)\,dx\right]$

$=\frac{7}{4}\left[\frac{4}{\pi}-\frac{1}{\pi^2}\int_{-1}^{1}30x\sin\pi x\,dx\right]=\frac{7}{4}\left[\frac{4}{\pi}-\frac{30}{\pi^2}\left(\frac{2}{\pi}+\frac{1}{\pi}\int_{-1}^{1}\cos\pi x\,dx\right)\right]$

$=\frac{7}{4}\left[\frac{4}{\pi}-\frac{60}{\pi^3}\right]=7(\pi^2-15)/\pi^3\approx-1.16$

$a_5=\frac{11}{2}\cdot\frac{1}{8}\int_{-1}^{1}(63x^5-70x^3+15x)\sin\pi x\,dx=\frac{11}{16\pi}\left[16+\int_{-1}^{1}(315x^4-210x^2+15)\cos\pi x\right]dx$

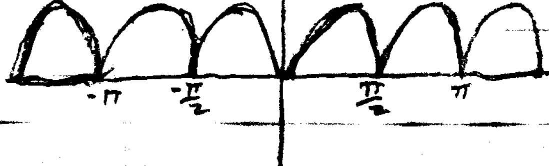
$=\frac{11}{16\pi}\left[16-\frac{1}{\pi}\int_{-1}^{1}(1260x^3-420x)\sin\pi x\,dx\right]=\frac{11}{16\pi}\left[16-\frac{1}{\pi^2}\left(1680+\int_{-1}^{1}(3780x^2-420)\cos\pi x\,dx\right)\right]$

$=\frac{11}{16\pi}\left[16-\frac{1}{\pi^2}\left(1680-\frac{1}{\pi}\int_{-1}^{1}7560x\sin\pi x\,dx\right)\right]=$

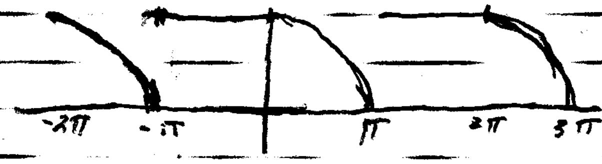
$\frac{11}{16\pi}\left[16-\frac{1}{\pi^2}\left(1680-\frac{15120}{\pi^2}\right)\right]\approx .219$

11.1 #2: $2\pi, 2\pi, \pi, \pi, 2, 2, 1, 1$

#10



#12



#16 $a_n=0$ since $f(x)$ is an odd function

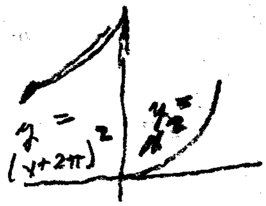
$b_n=\frac{1}{\pi}\int_{-\pi/2}^{\pi/2}x\sin nx\,dx=\frac{1}{\pi}\left[-\frac{x}{n}\cos nx\Big|_{-\pi/2}^{\pi/2}+\int_{-\pi/2}^{\pi/2}\frac{1}{n}\cos nx\,dx\right]$

$=-\cos(n\pi/2)/n+2\sin(n\pi/2)/(\pi n^2)=\begin{cases}2(-1)^k/(\pi(2k+1)^2) & \text{for } n=2k+1\\ -(-1)^k/(2k) & \text{for } n=2k\end{cases}$

So $\frac{2}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k\sin(2k+1)x}{(2k+1)^2}+\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\sin 2kx}{2k}$

$=\frac{2}{\pi}\sin x+\frac{1}{2}\sin 2x-\frac{2}{\pi 9}\sin 3x-\frac{1}{4}\sin 4x+\frac{2}{\pi 25}\sin 5x+\frac{1}{6}\sin 6x+\cdots$

#22



$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x^2\,dx = \frac{1}{2\pi}(2\pi)^3/3 = 4\pi^2/3$

$a_n = \frac{1}{\pi}\int_0^{2\pi} x^2\cos nx\,dx = \frac{1}{\pi}\left[x^2\frac{\sin nx}{n}\Big|_0^{2\pi} - \int_0^{2\pi} 2x\frac{\sin nx}{n}\,dx\right]$

$= \frac{2}{\pi n^2}\left[x\cos nx\Big|_0^{2\pi} - \int_0^{2\pi}\cos nx\,dx\right] = 4/n^2$

$b_n = \frac{1}{\pi}\int_0^{2\pi} x^2\sin nx\,dx = \frac{1}{\pi}\left[-x^2\frac{\cos nx}{n}\Big|_0^{2\pi} + \int_0^{2\pi} 2x\frac{\cos nx}{n}\,dx\right]$

$= \frac{1}{\pi}\left[-\frac{(2\pi)^2}{n} + \frac{2x\sin nx}{n^2}\Big|_0^{2\pi} - \int_0^{2\pi}\frac{2\sin nx}{n^2}\,dx\right] = -4\pi/n$

So $4\pi^2/3 + \sum_{n=1}^{+\infty} 4\cos nx/n^2 - \sum_{n=1}^{+\infty} 4\pi\sin nx/n$