314 hw 1

5.1

#2 $\sum_{m=0}^{+\infty} a_m m x^{m-1} + \sum_{m=0}^{+\infty} a_m x^{m+1} = 0$

$a_{s+2}(s+2) = -a_s$ for $s \geq -1$

$a_1 = 0$; $a_2 = -a_0/2$; $a_{2k+1} = 0$; $a_{2k} = (-1)^k a_0/(2^k k!)$

$y = a_0 \sum_{k=0}^{+\infty} (-1)^k x^{2k}/(2^k k!) = a_0 e^{-x^2/2}$

#6 $\sum_{m=0}^{+\infty} a_m m x^{m-1} + 3\sum_{m=0}^{+\infty} a_m x^m + 3\sum_{m=0}^{+\infty} a_m x^{m+2} = 0$

$a_{s+1}(s+1) + 3a_s + 3a_{s-2} = 0$ for $s \geq 0$

$a_1 = -3a_0$; $a_2 = -3a_1/2 = 9a_0/2$; $a_3 = -3(a_2 + a_0)/3 = -11a_0/2$

$a_4 = -3(a_3 + a_1)/4 = 51a_0/8$

$y = a_0(1 - 3x + 9x^2/2 - 11x^3/2 + 51x^4/8 + \dots)$

#10 $\sum_{m=0}^{+\infty} a_m m(m-1) x^{m-2} - \sum_{m=0}^{+\infty} a_m m x^m + \sum_{m=0}^{+\infty} a_m x^m = 0$

$a_{s+2}(s+2)(s+1) = a_s(s-1)$ for $s \geq 0$

$a_2 = -a_0/2$; $a_3 = 0$; $a_4 = a_2/(4 \cdot 3) = -a_0/4!$; $a_5 = 0$

$a_6 = 3a_4/(6 \cdot 5) = -3a_0/6!$; $a_8 = 5a_6/(8 \cdot 7) = -5 \cdot 3a_0/8!$

$y = a_1 x + a_0\left(1 - \frac{x^2}{2!} - \frac{x^4}{4!} - \sum_{k=3}^{+\infty} \frac{3 \cdots (2k-3)}{(2k)!} x^{2k}\right)$

5.2

#2 $\left| \frac{(-1)^{m+1}(x+1)^{2m+2}}{3^{m+1}(m+2)^2} \frac{3^m (m+1)^2}{(-1)^m (x+1)^{2m}} \right| = \frac{|x+1|^2 (1+\frac{1}{m})^2}{3(1+2/m)^2} \to \frac{|x+1|^2}{3} < 1$

for $|x+1| < \sqrt{3} = R$

#6 $\left| \frac{(-1)^{m+1} x^{2m+12} (m!)^2}{((m+1)!)^2 (-1)^m x^{2m+10}} \right| = \frac{|x|^2}{(m+1)^2} \to 0$ so $R = +\infty$

#10 $\left| \frac{(2(m+1))!\, x^{m+1}}{(m+1)^2} \frac{m^2}{(2m)!\, x^m} \right| = \frac{|x|(2m+2)(2m+1)}{(1+\frac{1}{m})^2} \to +\infty$ unless $x = 0$ so $R = 0$

#14 $m - 3 = s$, so $m = s+3$: $\sum_{s=0}^{+\infty} \frac{(-1)^{s+4}}{4^{s+3}} x^s = \sum_{s=0}^{+\infty} \frac{(-1)^s x^s}{4^{s+3}}$

$\left| \frac{(-1)^{s+5}}{4^{s+4}} \frac{x^{s+1}}{(-1)^{s+4}} \frac{4^{s+3}}{x^s} \right| = \frac{|x|}{4} \to \frac{|x|}{4} < 1$ for $|x| < 4 = R$

#18 $\sum_{m=0}^{+\infty} a_m m(m-1)x^{m-2} - \sum_{m=0}^{+\infty} a_m m x^{m-1} + \sum_{m=0}^{+\infty} a_m x^{m+1} = 0$

$a_{s+2}(s+2)(s+1) = a_{s+1}(s+1) - a_{s-1}$ for $s \geq 0$

$\therefore a_2 = a_1/2$; $a_3 = (2a_2 - a_0)/6 = a_1/6 - a_0/6$

$a_4 = (3a_3 - a_1)/12 = (-a_1/2 - a_0/2)/12 = -a_1/24 - a_0/24$

$a_5 = (4a_4 - a_2)/20 = (-a_1/6 - a_0/6 - a_1/2)/20 = -a_1/30 - a_0/120$

$\therefore y = a_0(1 - x^3/6 - x^4/24 - x^5/120 + \cdots) + a_1(x + x^2/2 + x^3/6 - x^4/24 - x^5/30 + \cdots)$

I don't think there is a simple (obvious) formula

#19 $\sum_{m=0}^{+\infty} a_m m(m-1)x^{m-2} + \sum_{m=0}^{+\infty} 4 a_m m x^m = 0$

$a_{s+2}(s+2)(s+1) = -4a_s s$ for $s \geq 0$

$a_2 \cdot 2 = -4a_0 0 \therefore a_2 = 0$; $a_3 = -4a_1/3!$

$a_4\, 4 \cdot 3 = -4a_2 2 = 0$. Likewise $a_{2k} = 0$ for $k \geq 1$

$a_5 = -4a_3 3/(5 \cdot 4) = (4)^2 3 a_1/5!$; $a_7 = -4^3 5 \cdot 3 a_1/7!$

$y = a_0 + a_1 \sum_{k=0}^{+\infty} \frac{(-1)^k 4^k (1 \cdot 3 \cdots 2k-1)}{(2k+1)!} x^{2k+1}$