5.8 314404

$(1-x^2)y''-2xy'+n(n+1)y=0$; $p(x)=1-x^2$; $q(x)=0$; $r(x)=1$
$\lambda=n(n+1)$

$\{P_m(x); m=0,1,\dots\}$ are orth on $[-1,1]$

$\|P_m\|^2=2/(2m+1)$; $a_m=\int_{-1}^{1} f(x)P_m(x)\,dx\,(2m+1)/2$

$P_0(x)=1$; $P_2(x)=\frac{1}{2}(3x^2-1)$; $P_4(x)=\frac{1}{8}(35x^4-30x^2+3)$

Problem #1: $f(x)=7x^4-6x^2$

Method 1: $a_0=\frac{1}{2}\int_{-1}^{1}(7x^4-6x^2)\,dx=\frac{1}{2}\left(\frac{14}{5}-4\right)=-.6$

$a_2=\frac{5}{2}\cdot\frac{1}{2}\int_{-1}^{1}(7x^4-6x^2)(3x^2-1)\,dx=\int_{-1}^{1}(21x^6-25x^4+6x^2)\,dx=0$

$a_4=\frac{9}{2}\frac{1}{8}\int_{-1}^{1}(7x^4-6x^2)(35x^4-30x^2+3)\,dx=$

$\frac{9}{16}\int_{-1}^{1}(245x^8-420x^6+201x^4-18x^2)\,dx=1.6$

Method 2: $7x^4-6x^2=\frac{a_4}{8}(35x^4-30x^2+3)+\frac{a_2}{2}(3x^2-1)+a_0$

$7=\frac{35}{8}a_4 \therefore a_4=1.6$; $-6=.2(-30)+\frac{3}{2}a_2 \therefore a_2=0$; $0=.6+a_0 \therefore a_0=-.6$

Problem #8: $f(x)=\cos\pi x$

$\int_{-1}^{1}\cos\pi x\, x^n\,dx=-\frac{1}{\pi}\int_{-1}^{1}nx^{n-1}\sin\pi x\,dx$

$\int_{-1}^{1}\sin\pi x\, x^n\,dx=(1-(-1)^n)/\pi+\frac{1}{\pi}\int_{-1}^{1}n\,x^{n-1}\cos\pi x\,dx$

$a_0=\frac{1}{2}\int_{-1}^{1}\cos\pi x\,dx=0$; $a_1=a_3=a_5=\dots=0$

$a_2=\frac{5}{2}\cdot\frac{1}{2}\int_{-1}^{1}\cos\pi x(3x^2-1)\,dx=\frac{-15}{4\pi}\int_{-1}^{1}\sin\pi x\;2x\,dx$

$=\frac{-15}{2\pi}\left[\frac{2}{\pi}+\frac{1}{\pi}\int_{-1}^{1}\cos\pi x\right]dx=\frac{-15}{\pi^2}\approx-1.52$

$a_4=\frac{9}{2}\cdot\frac{1}{8}\int_{-1}^{1}\cos\pi x(35x^4-30x^2+3)\,dx=$

$-\frac{9}{16\pi}\int_{-1}^{1}\sin\pi x(140x^3-60x)\,dx=$

$-\frac{9}{16\pi^2}\left[280-120+\int_{-1}^{1}\cos\pi x(420x^2-60)\,dx\right]=$

$-\frac{9}{16\pi^2}\left[160-\frac{840}{\pi}\int_{-1}^{1}(\sin\pi x)\,x\,dx\right]=$

$-\frac{9}{16\pi^2}\left[160-\frac{840}{\pi}\cdot\frac{2}{\pi}\right]\approx.59$

getting higher order terms by hand is rather tedious