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#4 $y'' - y = 0$

$$\sum_{m=0}^{+\infty} a_m m(m-1)x^{m-2} - \sum_{m=0}^{+\infty} a_m x^m = 0$$

$$\sum_{a=0}^{+\infty} a_{a+2}(a+2)(a+1)x^a - \sum_{a=0}^{+\infty} a_a x^a = 0$$

$\therefore a_{a+2}(a+2)(a+1) = a_a$ for $a \geq 0$

$a_{2k} = a_0/(2k)!$; $a_{2k+1} = a_1/(2k+1)!$

$$\therefore y = a_0 \sum_{k=0}^{+\infty} \frac{x^{2k}}{(2k)!} + a_1 \sum_{k=0}^{+\infty} \frac{x^{2k+1}}{(2k+1)!} = a_0(e^x + e^{-x})/2 + a_1(e^x - e^{-x})/2$$

$= a_0 \cosh x + a_1 \sinh x$

#8 $(8x^5 + 4x^3)y' = (5x^4 + 12x^2)y$

$$\sum_{m=0}^{+\infty} a_m m x^{m+4} + \sum_{m=0}^{+\infty} 4a_m m x^{m+2} = \sum_{m=0}^{+\infty} 5a_m x^{m+4} + \sum_{m=0}^{+\infty} 12 a_m x^{m+2}$$

$$\sum_{a=0}^{+\infty} a_a a x^{a+4} + \sum_{a=-2}^{+\infty} 4a_{a+2}(a+2)x^{a+4} = \sum_{a=0}^{+\infty} 5a_a x^{a+4} + \sum_{a=-2}^{+\infty} 12 a_{a+2} x^{a+4}$$

$\therefore 0 = 12a_0$, or $a_0 = 0$; $4a_1 = 12a_1$, or $a_1 = 0$

$a_{a+2}(a-1)4 = a_a(5-a)$ for $a \geq 0$

$-4a_2 = 5a_0 \therefore a_2 = 0$; $a_3 0 = 4a_1$; $a_4 4 = 5a_2 = 0$

$a_5 8 = 2a_3 \therefore a_5 = a_3/4$; $a_6 12 = a_4 = 0$; $a_7 16 = a_5 0 = 0$

$a_m = 0$ for $m \geq 8$ $\therefore y = a_3(x^3 + x^5/4)$

#16 $(1-x^2)y'' - 2xy' + 30y$; $y(0) = 0$; $y'(0) = \frac{15}{8}$ Find $y(0.5)$

$$\sum_{m=0}^{+\infty} a_m m(m-1)x^{m-2} - \sum_{m=0}^{+\infty} a_m m(m-1)x^m - \sum_{m=0}^{+\infty} 2a_m m x^m + \sum_{m=0}^{+\infty} 30 a_m x^m = 0$$

$a_{m+2}(m+2)(m+1) = a_m(m^2 + m - 30) =$ for $m \geq 0$

$a_0 = y(0) = 0$; $a_1 = y'(0) = \frac{15}{8}$; $a_2 2 = -30a_0 = 0$

$a_3 6 = -28a_1 \therefore a_3 = -35/4$; $a_4 12 = -24a_2 = 0$; $a_5 20 = -18a_3$

$\therefore a_5 = 63/8$; $42a_7 = a_5 \cdot 0$; $a_m = 0$ for $m \geq 8$

$\therefore y = \frac{15}{8}x - \frac{35}{4}x^3 + \frac{63}{8}x^5$; $y(.5) = 0.089844$

5.2

#16 $y'' + xy = 0$

$$\sum_{m=0}^{+\infty} a_m m(m-1)x^{m-2} + \sum_{m=0}^{+\infty} a_m x^{m+1} = 0$$

$a_{k+3}(k+3)(k+2) = -a_k$ for $k \geq -1$

$a_2 = 0$; $a_3 = \frac{-a_0}{3\cdot 2}$; $a_4 = \frac{-a_1}{4\cdot 3}$; $a_5 = 0$; $a_6 = \frac{-a_3}{6\cdot 5} = \frac{a_0}{6\cdot 5\cdot 3\cdot 2}$;

$a_7 = \frac{-a_4}{7\cdot 6} = \frac{a_1}{7\cdot 6\cdot 4\cdot 3}$; $a_8 = \frac{-a_5}{8\cdot 7} = 0$

$$y = a_0 \sum_{k=0}^{+\infty} \frac{(-1)^k x^{3k}}{2\cdot 3\cdot 5\cdot 6\cdots(3k-1)3k} + a_1 \sum_{k=0}^{+\infty} \frac{(-1)^k x^{3k+1}}{3\cdot 4\cdot 6\cdot 7\cdots(3k)(3k+1)}$$

#22 $y'' - 4xy' + (4x^2 - 2)y = 0$

$$\sum_{m=0}^{+\infty} a_m m(m-1)x^{m-2} - \sum_{m=0}^{+\infty} a_m m x^m + \sum_{m=0}^{+\infty} 4a_m x^{m+2} - 2\sum_{m=0}^{+\infty} a_m x^m = 0$$

$a_{k+2}(k+2)(k+1) - a_k 4k + 4a_{k-2} - 2a_k = 0$ for $k \geq 0$

$a_{k+2}(k+2)(k+1) = a_k(4k+2) - 4a_{k-2}$

$a_2 = a_0$; $a_3 = a_1$; $a_4 = (10a_2 - 4a_0)/12 = a_0/2$

$a_5 = (14a_3 - 4a_1)/20 = a_1/2$; $a_6 = (18a_4 - 4a_2)/30 = a_0/6$

$a_7 = (22a_5 - 4a_3)/42 = a_1/6$

For $k = 2k$, $a_{2k} = a_0/k!$. For $k = 2k+1$, $a_{2k+1} = a_1/k!$

$$\therefore y = a_0 \sum_{k=0}^{+\infty} \frac{x^{2k}}{k!} + a_1 x \sum_{k=0}^{+\infty} \frac{x^{2k}}{k!} = a_0 e^{x^2} + a_1 x e^{x^2}$$

#8 $$\left| \frac{(4m+4)!\, x^{m+1}(m!)^4}{((m+1)!)^4\, (4m)!\, x^m} \right| = \frac{(4m+4)(4m+3)(4m+2)(4m+1)\,|x|}{(m+1)^4}$$

$\to 256|x| < 1$ iff $|x| < \frac{1}{256} = R$

#12 $$\left| \frac{(m+2)(m+1)\, x^{2m+3}(2m+1)!}{(2m+3)!\,(m+1)(m)\, x^{2m+1}} \right| = \frac{(m+2)|x|^2}{(2m+3)(2m+2)} \to 0$$

$\therefore R = +\infty$