13.6 | 313 Homework 9

#12 a) $-i\sinh(-\pi+2i) = \frac{-i}{2}\left[e^{-\pi}(\cos 2+i\sin 2)-e^{\pi}(\cos 2-i\sin 2)\right]$

$= \frac{\sin 2}{2}(e^{\pi}+e^{-\pi}) + i\frac{\cos 2}{2}(e^{\pi}-e^{-\pi}) = \sin 2\cosh\pi + i\cos 2\sinh\pi$

b) By (14) $\sin(2+\pi i) = -i\sinh(i(2+\pi i)) = -i\sinh(-\pi+2i)$ ″

#18 $\arcsin(100) = \frac{1}{i}\ln\left[100i+\sqrt{1-10^4}\right] = \frac{1}{i}\ln\left[100i \pm i\sqrt{10^4-1}\right]$

$= \frac{1}{i}\left[\ln(100\pm\sqrt{10^4-1}) + (\frac{\pi}{2}+2\pi k)i\right] = (\frac{\pi}{2}+2\pi k) - i\ln(100\pm\sqrt{10^4-1})$

OR $100 = \sin z = \sin x\cosh y + i\cos x\sinh y$

So $0=\cos x\sinh y$. If $y=0$, then $\cosh y=1$ and $\sin x=100$ *

If $\cos x=0$, then $x=\frac{\pi}{2}+n\pi$. If n is odd $\sin x=-1$ and $\cosh y=-100$ *

So $x=\frac{\pi}{2}+2k\pi$. Thus $\cosh y=100=\frac{1}{2}(e^{y}+e^{-y})$

so $e^{2y}-200e^{y}+1=0$ so $e^{y}=(200\pm\sqrt{4\cdot 10^4-4})/2$

$=100\pm\sqrt{10^4-1}$ ∴ $y=\ln(100\pm\sqrt{10^4-1})$ as above

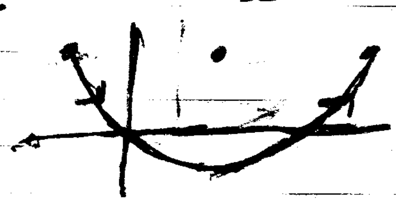
13.7 #2: $\mathrm{Ln}(2+2i)=\frac{1}{2}\ln 8+\frac{\pi}{4}i$ #6: $\mathrm{Ln}(-100)=\ln 100+\pi i$

#12: $\ln(e)=1+2\pi k i$ #14 $\ln(4+3i)=\ln 5+(\mathrm{Arctan}\frac{3}{4}+2\pi k)i$

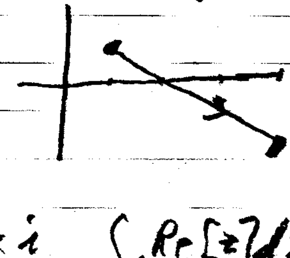
#20 $\ln z=e-\pi i$ iff $z=e^{e-\pi i}=e^{e}(\cos(-\pi)+i\sin(-\pi))=-e^{e}$

#26 $\mathrm{Ln}(-1)=\pi i$; $(1-2i)\pi i=2\pi-\pi i$; $e^{2\pi-\pi i}=-e^{2\pi}$

14.1 #6: $x=3+5\cos t$ $\quad(x-3)^2+(y-4)^2=25$
$y=4+5\sin t$ $\quad -2\le x\le 8$; $-1\le y\le 4$



#10 $(4-2i)-(1+i)=3-3i$
$x=1+3t$; $y=1-3t$
$0\le t\le 1$



#20 $z(x)=x+x^2 i$; $z'=1+2xi$ $\quad \int_C \mathrm{Re}[z]\,dz=\int_0^1 x(1+2xi)\,dx=\frac{1}{2}+\frac{2}{3}i$

#24 $\int(z+\frac{1}{z})\,dz=\frac{z^2}{2}+\ln z$ on slit plane at $z\to -1$, $\ln z\to\pm\pi i$ from upper and lower half planes, So $\oint_C(z+\frac{1}{z})\,dz=\left[\frac{z^2}{2}+\ln z\right]_{-1}^{-1}=\boxed{2\pi i}$