13.4

#6 No. f is real valued, not constant; so can't be

#8 No. Arg z is not cont. on neg. real axis, so can't be differentiable. It is everywhere else

#16 $v_x = \frac{x}{x^2+y^2}$; $v_{xx} = \frac{y^2-x^2}{(x^2+y^2)^2}$; $v_y = \frac{y}{(x^2+y^2)}$; $v_{yy} = \frac{x^2-y^2}{(x^2+y^2)^2}$

So $\nabla^2 v \equiv 0$ ∴ harmonic. To find u, use polar coord

$v = \ln r$; $v_r = \frac{1}{r}$, $v_\theta = 0$ ∴ $u_r = v_\theta / r = 0$, $u_\theta = -r v_r = -1$

∴ $u = -\theta = -\arg z$ However, cont. branch of arg z D.N.E.

#18 $u = r^{-2}$ ∴ $u_r = -2r^{-3}$, $u_{rr} = 6r^{-4}$, $u_\theta = u_{\theta\theta} = 0$

$\nabla^2 u = u_{rr} + u_{\theta\theta}/r^2 + u_r/r = 6r^{-4} - 2u^{-4} \neq 0$ ∴ not harmonic

#22 $u_{xx} = 9e^{3x}\cos ay$; $u_{yy} = -a^2 e^{3x}\cos ay$; $\nabla u = 0$ iff $a = \pm 3$

$v_y = 3e^{3x}\cos 3y$; $v_x = 3e^{3x}\sin 3y$; $v = e^{3x}\sin 3y + g(y)$

$v_y = 3e^{3x}\cos 3y + g'(y) = 3e^{3x}\cos 3y$ ∴ $g' = 0$ $v = e^{3x}\sin 3y$

NOTE $u + iv = e^{3z}$

13.5

#2. $e^z = -e^3$; $|e^z| = e^3$ #8. $e^z = i$; $|e^z| = 1$

#12. $e^{1/z} = e^{(x-iy)/(x^2+y^2)} = e^{\frac{x}{x^2+y^2}}\cos\left(\frac{y}{x^2+y^2}\right) - i e^{\frac{x}{x^2+y^2}}\sin\left(\frac{y}{x^2+y^2}\right)$

#14 $|1+i| = \sqrt{2}$; $\operatorname{Arg}(1+i) = \frac{\pi}{4}$ ∴ $1+i = \sqrt{2}\, e^{i\pi/4}$

#18 $e^{3z} = 4$ iff $e^{3x} = 4$ and $3y = \arg 4 = 2\pi k i$

∴ $z = \frac{1}{3}\ln 4 + 2\pi k i/3$ where k is integer