313 homework 7

13.1 #4 First note that $zw = z(w+0) = zw + z0$
subtract zw from both sides and thus $0 = z0$
Now assume $zw = 0$. If $z = 0$, then we are done.
If $z \neq 0$, then z^{-1} exists, so $w = z^{-1}zw = z^{-1}0 = 0$ by above

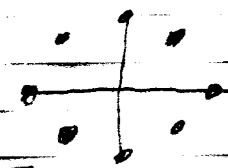
#10 $Re(z_2^2) = Re[-9-20i] = \boxed{-9}$; $(Re\, z_2)^2 = 4^4 = \boxed{16}$

#16 $I_m(z^3) = I_m(x^3 - 3xy^2 + i(3x^2y - y^3)) = 3x^2 - y^3$; $(I_m z)^3 = y^3$

13.2 #8 $\frac{2+3i}{5+4i} = \frac{(2+3i)(5-4i)}{41} = \frac{22}{41} + \frac{7}{41}i = \boxed{\frac{\sqrt{533}}{41} \operatorname{cis}\left[\operatorname{Arctan} \frac{7}{22}\right]}$

#14 $\arg(1+i) = \frac{\pi}{4} + 2\pi k$ so $\arg(1+i)^{12} = 3\pi + 2\pi k$, so $\operatorname{Arg}(1+i)^{12} = \boxed{\pi}$

#22 $\sqrt[8]{1} = \operatorname{cis}\left(\frac{2k\pi}{8}\right)$ where $k = 0, \ldots, 7$ so get in order:
$1, (1+i)/\sqrt{2}, i, (-1+i)/\sqrt{2}, -1, (-1-i)/\sqrt{2}, -i, (1-i)/\sqrt{2}$



#28 $z^2 = \frac{14i - 5 + \sqrt{(5+14i)^2 + 4(24+10i)}}{2} = \frac{14i - 5 + \sqrt{-75-100i}}{2}$

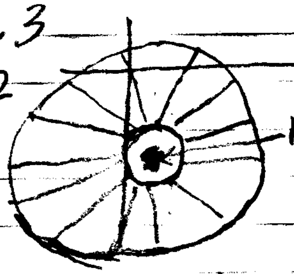
$-75 - 100i = 125\left(-\frac{3}{5} - \frac{4}{5}i\right) = 125 \operatorname{cis}\theta$ where $\cos\theta = -\frac{3}{4}$, $\sin\theta = -\frac{3}{5}$

$\therefore \sqrt{-75-100i} = 5\sqrt{5} \operatorname{cis}\left(\frac{\theta}{2}\right)$ where $\cos^2\left(\frac{\theta}{2}\right) = \frac{1}{2}[1 + \cos\theta] = \frac{1}{5}$ and

$\sin^2\theta = \frac{1}{2}[1 - \cos\theta] = \frac{4}{5}$ so $\sqrt{-75-100i} = 5(-1+2i)$ and $5(1-2i)$

So $z^2 = -5 + 12i$ and $2i$ Hence $z = \pm\sqrt{13}\left[\frac{2}{\sqrt{13}} + \frac{3i}{\sqrt{13}}\right] = \boxed{\pm(2+3i) \text{ and } \pm(1+i)}$

13.3 #2



$1-4i$

#14 $f = \frac{1-\bar{z}}{|1-z|^2} = \frac{1-x}{(1-x)^2+y^2} + \frac{yi}{(1-x)^2+y^2}$

$f\left(\frac{1}{2} + \frac{1}{4}i\right) = (1/2)/(5/16) + i\left(\frac{1}{4}\right)/(5/16)^{?} = \boxed{8/5 + i\, 4/5}$

#16 $Re(x^2)/|x^2| = 1$; $Re[(iy)^2]/|(iy)^2| = -y^2/y^2 = -1$. So limit
from real and imag axes are different. so limit DNE

#24 $\frac{(z+i)^2 2z - z^2(z+i)2}{(z+i)^4} = \boxed{2zi/(z+i)^3}$
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