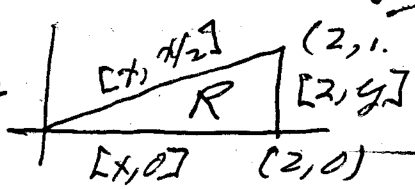
10.4 313 Homework

#4 $\iint_R (e^x+e^y)\,dA = \int_0^2\int_0^{x/2}(e^x+e^y)\,dy\,dx$



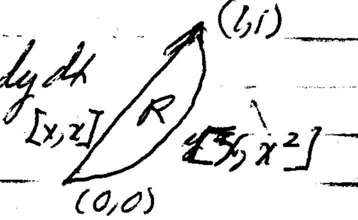
$= \int_0^2 (xe^x/2 + e^{x/2} - 1)\,dx = [e^x(x-1)/2 + 2e^{x/2} - x]_0^2$

$= e^2/2 + 2e - 2 + \frac12 - 2 = \boxed{e^2/2+2e-\frac72}$

$\oint_C(-e^y\,dx + e^x\,dy) = \int_0^2 -dx + \int_0^1 e^2\,dy - \int_0^2(-e^{x/2}+e^x/2)\,dx$

$= -2 + e^2 - (-2e + e^2/2 + 2 - \frac12) = \boxed{e^2/2+2e-\frac72}$

#6 $\iint_R (2x\sinh y - x\sinh y)\,dA = \int_0^1\int_{x^2}^{x}(x\sinh y)\,dy\,dx$



$= \int_0^1 (x\cosh x - x\cosh x^2)\,dx =$

$[x\sinh x - \cosh x - \frac12\sinh x^2]_0^1 = \sinh 1 - \cosh 1 - \frac12\sinh 1 + 1$

$= \boxed{\frac12\sinh 1 - \cosh 1 + 1}$

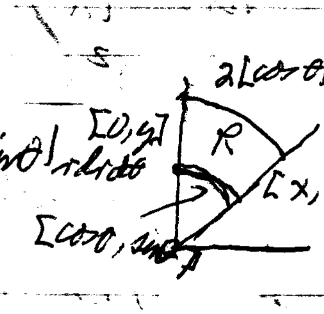
$\oint_C(x\cosh y\,dx + x^2\sinh y\,dy) = \int_0^1(x\cosh x^2 + x^2\sinh x^2\cdot 2x)\,dx -$

$\int_0^1(x\cosh x\ \cancel{+}\ x^2\sinh x)\,dx = [\frac12\sinh x^2 + x^2\cosh x^2 - \sinh x^2]_0^1 -$

$[x^2\cosh x - x\sinh x + \cosh x]_0^1 = \frac12\sinh 1 + \cosh 1 - \sinh 1 - (\cosh 1 - \sinh 1 + \cosh 1 - 1)$

$= \boxed{\frac12\sinh 1 - \cosh 1 + 1}$

#12 $\iint_R\left[\frac{-1}{y^2} - 2x^2y\right]dA = -\int_{\pi/4}^{\pi/2}\int_1^2\left(\frac{1}{r^2\sin^2\theta} + 2r^3\cos^2\theta\sin\theta\right)r\,dr\,d\theta$



$= -\int_{\pi/4}^{\pi/2}\left(\frac{\ln 2}{\sin^2\theta} + \frac{62}{5}\cos^2\theta\sin\theta\right)d\theta =$

$+\left[\ln 2\cot\theta + \frac{62}{5}\cdot\frac13\cos^3\theta\right]_{\pi/4}^{\pi/2} = \boxed{-\ln 2 - 31/15\sqrt2}$

$\oint_C(x^2y^2\,dx - x/y^2\,dy) = -\int_{\pi/4}^{\pi/2}(-\cos^2\theta\sin^3\theta - \cos^2\theta/\sin^2\theta)\,d\theta +$

$\int_{1/\sqrt2}^{\sqrt2}(x^4 - \frac1x)\,dx + \int_{\pi/4}^{\pi/2}(-2^5\cos^2\theta\sin^3\theta - \cos^2\theta/\sin^2\theta)\,d\theta + 0$

$= \int_{1/\sqrt2}^{\sqrt2}(x^4-\frac1x)\,dx - 31\int_{\pi/4}^{\pi/2}(\cos^2\theta\sin^3\theta)\,d\theta = \frac{31}{20\sqrt2} - \ln 2 - 31\left[\frac{\cos^5\theta}{5} - \frac{\cos^3\theta}{3}\right]_{\pi/4}^{\pi/2}$

$= \frac{31}{20\sqrt2} - \ln 2 + \frac{31}{5\cdot4\sqrt2} - \frac{31}{3\cdot2\sqrt2} = \boxed{-\ln 2 - 31/15\sqrt2}$

#2 This is surface $z=0$; $u=$ const. is circle around origin
$v=$ const. is radial line; $\vec{N} = \vec{r}_u \times \vec{r}_v = u\vec{k}$
$\nabla(z) = [0,0,1]$; geometrically: normal to x-y plane is in $\vec{k}$ direct.

#6 $x^2 - 16y^2 = 16u^2\cosh^2 v - 16u^2 \sinh^2 v = 16u^2 = 16z$
This looks like a saddle. If $u=k$, then $x^2-16y^2=16k^2$
so get a hyperbola. If $v=k$, then get a parabola
For example, $v=0$ gives $x^2=16z$;
$\vec{N} = [4\cosh v, \sinh v, 2u] \times [4u\sinh v, u\cosh v, 0] =$
$[-2u^2\cosh v, 8u^2\sinh v, 4u] = 4u[\frac{-x}{8}, 2y, 1] = 4u\,\nabla(z - \frac{x^2}{16} + y^2)$

#12 $y = 30 - 5x + 3z$, so $\vec{r} = [x, 30-5x+3z, z]$
a normal is $\nabla(5x+y-3z) = [5, 1, -3]$

#14 $x-1 = 5(\cos\theta\sin\phi)$; $y+2 = 5\sin\theta\sin\phi$; $z = 5\cos\phi$
so $\vec{r} = [1+5\cos\theta\sin\phi, -2+5\sin\theta\sin\phi, 5\cos\phi]$ for $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$
OR $z = \sqrt{25-(x-1)^2-(y+2)^2}$ and $z = -\sqrt{25-(x-1)^2-(y+2)^2}$
for $(x-1)^2+(y+2)^2 \le 25$ for top + bottom respectively
a normal is $[2(x-1), 2(y+2), 2z]$

#16 $\vec{r} = [x, y, 4x^2+y^2]$ OR $\vec{r} = [u\cos v, 2u\sin v, 4u^2]$
a normal is $[-8x, -2y, 1]$

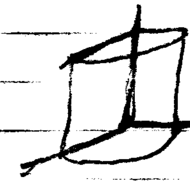
#2 Let $z=4-x-y$ for $x+y\le 4$; $0\le x$; $0\le y$; $\iint_S z^2\cos\gamma\,dS =$

$\int_0^4\int_0^{4-x}(4-x-y)^2\,dy\,dx = \int_0^4 \frac{1}{3}(4-x)^3\,dx = \frac{1}{12}4^4 = \frac{64}{3}$

The other 2 contributions are the same so $3\cdot\frac{64}{3} = \boxed{64}$

#4 Let $x=\sqrt{9-y^2}$ for $0\le y\le 3$; $0\le z\le 2$

Then $\iint_S e^y\cos\alpha\,dS = \int_0^3\int_0^2 e^y\,dz\,dy = 2(e^3-1)$



Let $y=\sqrt{9-x^2}$ for $0\le x\le 3$, $0\le z\le 2$

Then $\iint_S(-e^z)\cos\beta\,dS = \int_0^3\int_0^2(-e^z)\,dz\,dy = 3(1-e^2)$

$\iint_S e^x\cos\gamma\,dS=0$ so $\boxed{2e^3-3e^2+1}$

#14 Let $z=2-x-y$; $x+y\le 2$; $0\le x$; $0\le y$; $\vec{N}=[1,1,1]$; $|\vec{N}|=\sqrt{3}$

$\iint_S(\cos y+\sin x)\,dS = \int_0^2\int_0^{2-x}(\cos y+\sin x)\sqrt{3}\,dy\,dx =$

$\sqrt{3}\int_0^2(\sin(2-x)+(2-x)\sin x)\,dx = \sqrt{3}\left[\cos(2-x)-2\cos x+x\cos x-\sin x\right]_0^2$

$=\sqrt{3}[1-2\cos 2+2\cos 2-\sin 2-\cos 2+2]$

$=\boxed{\sqrt{3}[3-\sin 2-\cos 2]}$

#16 Let $y=\sqrt{16-x^2}$; $-4\le x\le 4$; $0\le z\le 4$; $\vec{N}=\left[\frac{-x}{\sqrt{16-x^2}},1,0\right]$

$|\vec{N}|=\frac{4}{\sqrt{16-x^2}}$; $\iint_S(ye^x+xe^y+e^z)\,dS=\int_0^4\int_{-4}^4\left(4e^x+\frac{4xe^{\sqrt{16-x^2}}}{\sqrt{16-x^2}}+\frac{4e^z}{\sqrt{16-x^2}}\right)dx\,dz$

$=\int_0^4\left[4(e^4-e^{-4})+e^{\sqrt{16-x^2}}\Big|_{-4}^4+4e^z\pi\right]dz=\boxed{16(e^4-e^{-4})+0+4\pi(e^4-1)}$

since $\int_{-4}^4\frac{1}{\sqrt{16-x^2}}\,dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{\sqrt{16-16\sin^2\theta}}4\cos\theta\,d\theta=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}1\,d\theta=\pi$

OR let $\vec{r}=[4\cos u, 4\sin u, v]$; $\vec{N}=[4\cos u, 4\sin u, 0]$

$|\vec{N}|=4$ so $\int_0^4\int_0^\pi\left[4\sin u\,e^{4\cos u}+4\cos u\,e^{4\sin u}+e^v\right]4\,du\,dv$

$=4\int_0^4\left(-e^{4\cos u}+e^{4\sin u}+ue^v\right]_0^\pi dv = 4\int_0^4(e^4-e^{-4}+\pi e^v)\,dv$

$=\boxed{16(e^4-e^{-4})+4\pi(e^4-1)}$