10.1

313 homework 4

#2 $\vec r(t) = [t, 10t]$ for $0 \le t \le 2$; $\vec r\,'(t) = [1, 10]$

$\int_C \vec F(\vec r)\cdot d\vec r = \int_0^2 [1000t^3, t^3]\cdot[1,10]\,dt = \int_0^2 1010\,t^3\,dt = \boxed{4040}$

Note that shortest path does not require least work.

#8 $\int_0^{\frac12}[\cosh t, \sinh t^2, e^{t^3}]\cdot[1, 2t, 3t^2]\,dt =$

$\int_0^{\frac12}(\cosh t + \sinh t^2\, 2t + e^{t^3}3t^2)\,dt = [\sinh t + \cosh t^2 + e^{t^3}]_0^{\frac12}$

$= \sinh\frac12 + \cosh\frac14 - 1 + e^{\frac18} - 1 = \boxed{\sinh\frac12 + \cosh\frac14 + e^{\frac18} - 2}$

#18 $\int_0^{\frac{\pi}{4}}[\cos t\sin t, \tan t, 0]\,dt = [\frac12\sin^2 t, \ln|\sec t|, 0]_0^{\frac{\pi}{4}} =$

$\boxed{[\frac14, \ln\sqrt2, 0]}$

10.2

#6 Let $f(x,y,z) = \frac12 e^{x^2+y^2-2z}$. Then $\nabla f = [x, y, -1]\,e^{x^2+y^2-2z} = \vec F$

So $\int_{(0,0,0)}^{(1,1,0)}\vec F\cdot d\vec r = \frac12 e^{x^2+y^2-2z}\Big|_{(0,0,0)}^{(1,1,0)} = \boxed{\frac12(e^2-1)}$

#12 Let $f(x,y,z) = x^3e^{2y} + x^2/2$ so $\nabla f = \vec F$

So $\int_{(0,0,0)}^{(a,b,c)}\vec F\cdot d\vec r = \boxed{a^3e^{2b} + a^2/2}$

10.3

#2

$\int_0^1\int_x^{2x}(x+y)^2\,dy\,dx = \int_0^1\frac13(x+y)^3\Big|_x^{2x}dx = \int_0^1\frac13(19x^3)\,dx$

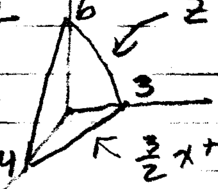
$= \frac{19}{12}x^4\Big|_0^1 = \boxed{\frac{19}{12}}$

Check by reversing order $\int_0^1\int_{\frac{y}{2}}^{y}(x+y)^2\,dx\,dy + \int_1^2\int_{\frac{y}{2}}^{1}(x+y)^2\,dx\,dy$

$= \frac13\int_0^1\frac{37}{8}y^3\,dy + \frac13\int_1^2[(y+1)^3 - \frac{27}{8}y^3]\,dy = \frac{37}{96} + \frac1{12}(65 - \frac{27}{8}\cdot 15)$

$= (37 + 520 - 405)/96 = 152/96 = 19/12$

#12



$z = 6 - \frac32 x - 2y$

$\frac32 x + 2y = 6$

$\int_0^4\int_0^{3-3x/4}(6 - \frac32 x - 2y)\,dy\,dx =$

$\int_0^4[6(3-3x/4) - \frac{3x}{2}(3-3x/4) - (3-3x/4)^2]\,dx$

$= \int_0^4(9 - \frac92 x + \frac{9}{16}x^2)\,dx = 9(4 - 4 + \frac43) = \boxed{12}$

CHECK: for parallelepiped $\begin{vmatrix}4&0&0\\0&3&0\\0&0&6\end{vmatrix} = 72$ Take $\frac16$ for tetrahedron and get $72/6 = 12$