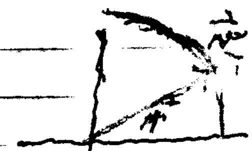
9.6 #6: $\frac{\delta w}{\delta x}\frac{\delta x}{\delta u} + \frac{\delta w}{\delta y}\frac{\delta y}{\delta u} = 8(u+2v)\cdot 1 - 8(2u-v)\cdot 2 = -24u+32v =$

$\frac{\delta w}{\delta u} = \frac{\delta}{\delta u}\left[4(u+2v)^2 - 4(2u-v)^2\right] = \frac{\delta}{\delta u}\left[-12u^2+32uv+12v^2\right]$

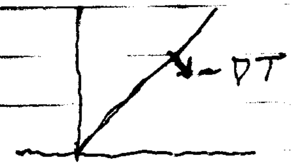
9.7

#8 $\nabla f = \left[\frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}\right]\Big|_{(4,3)} = \left[\frac{8}{25}, \frac{6}{25}\right] = \vec v$



level curves are circles, f increasing with increasing radius, so velocity is in direction of pos. vect

#14 $\nabla T = [-y, x]/(x^2+y^2)\big|_{(2,2)} = [-\frac{1}{4}, \frac{1}{4}]$



level curves are radial lines, T increasing in counterclock dir so flow is $\perp$ to line $y = x$

#36 $\nabla f = -[x,y,z]/(x^2+y^2+z^2)^{3/2}\big|_{(1,2,-2)} = -[4,2,-4]/216$

$|\vec a| = 3$; $\vec b = [1,2,-2]/3$; $D_{\vec a} f = \nabla f \cdot \vec b = -16/(3\cdot 216) = -2/(27\cdot 3) = \boxed{-2/81}$

#40 $f_x = ye^x \therefore f = ye^x + g(y,z) \therefore f_y = e^x = e^x + g_y \therefore g(y,z) = h(z)$

$\therefore f_z = 2z = h'(z) \therefore h(z) = z^2 + c$ Hence $f = ye^x + z^2 + c$

Check that $f_x = ye^x$; $f_y = e^x$; $f_z = 2z$

9.8

#4 $\nabla\cdot\vec v \equiv 0$ Note: $\vec v = \nabla\left(-(x^2+y^2+z^2)^{-1/2}\right)$ So this says

$\nabla^2 f = \nabla\cdot\nabla f \equiv 0$ where $f = -(x^2+y^2+z^2)^{-1/2}$

This does not always happen

#18 $\nabla f = [-y, x]/(x^2+y^2)$; $\nabla\cdot\nabla f = (2xy - 2yx)/(x^2+y^2)^2 \equiv 0$

9.9

#4 As in 9.8 #4, $\vec v = \nabla f$ so $\nabla\times\vec v = \nabla\times\nabla f = \vec 0$. This does always happen

Can check by direct calculation

#18 $\operatorname{curl}(f\vec a) = \begin{vmatrix} \vec i & \vec j & \vec k \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ xy^3z & xyz^3 & x^3yz \end{vmatrix} = \vec i(x^3z - 3xyz^2) + \vec j(xy^3 - 3x^2yz) + \vec k(yz^3 - 3xy^2z)$

CHECK $\nabla f \times \vec v = [xz(x^2 - zy), xy(y^2 - zx), yz(z^2 - xy)]$

f curl $\vec v$ $= [-2z, -2x, -2y]$ and $\nabla f \times \vec v + f \operatorname{curl}\vec v$ gives same answer