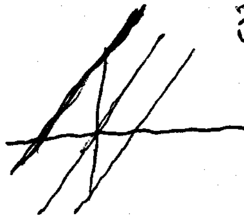
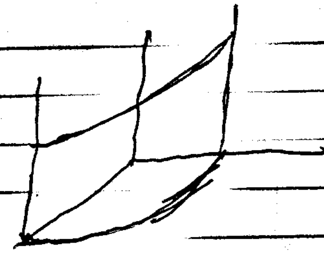
313 homework 2

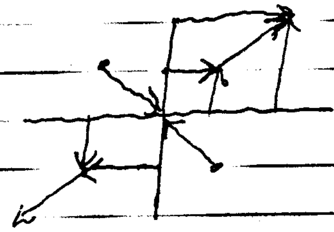
9.4 #2 $T = 4x - 3y = c$
give parallel lines



9.4 #10 $f = x^2 + 4y^2 = c$
gives elliptical cylinders
parallel to z axis



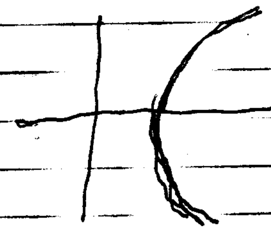
9.4 #16



9.5 #2 $\vec{b} = [11-5, 3-1, 0-2] = [6, 2, -2]$ so $\vec{q}(w) = [5+6w, 1+2w, 2-2w]$

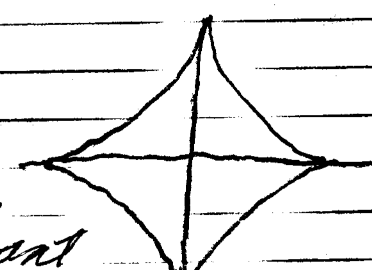
9.5 #6 $\vec{r}(t) = [\cos t, \sin t, \sin t]$

9.5 #16 $x^2 - y^2 = \cosh^2 t - \sinh^2 t = 1$
so right half of hyperbola in x, y plane
That's why they are called
hyperbolic functions



9.5 #24 $\vec{r}(t) = [3\cos t, \sin 3t, 4t]$
$\vec{r}\,'(t) = [-3\sin t, 3\cos t, 4]$
$|\vec{r}\,'(t)| = 5$; $\vec{u}(t) = \frac{1}{5}[-3\sin t, 3\cos t, 4]$
$P = (3, 0, 8\pi)$; $t = 2\pi$; $\vec{r}\,'(2\pi) = [0, 3, 4]$
$\vec{q}(w) = [3, 3w, 8\pi + 4w]$ like figure 202

9.5 #28 a as shown on the handout



$\frac{ds}{dt} = |\vec{r}\,'(t)| = 3a|\cos t \sin t|$ for $0 \le t \le 2\pi$

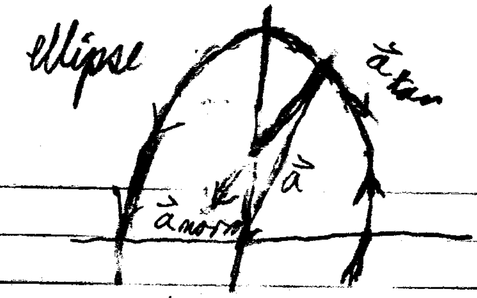
By symmetry find length in 1st quadrant
and multiply by 4

$l = \int_0^{2\pi} 3a|\cos t \sin t|\,dt = 12a\int_0^{\frac{\pi}{2}} \cos t \sin t\,dt$

$= 12a\left[\frac{\sin^2 t}{2}\right]_0^{\frac{\pi}{2}} = 6a$

Note length is a little less than length of a circle
of radius a

homework 2 cont



9.5#34 $\vec{r}(t) = [\cos t, 2\sin t, 0]$ is pos.

$\vec{v} = \vec{r}\,'(t) = [-\sin t, 2\cos t, 0]$ is vel.

$\vec{a} = \vec{r}\,''(t) = [-\cos t, -2\sin t, 0]$ is acc

$\frac{ds}{dt} = |\vec{r}\,'(t)| = \sqrt{\sin^2 t + 4\cos^2 t}$ is speed

$\vec{v}\cdot\vec{a} = -3\cos t\sin t$; $\vec{a}_{tan} = -3\cos t\sin t[-\sin t, 2\cos t, 0]/(\sin^2 t + 4\cos^2 t)$

$\vec{a}_{norm} = \vec{a} - \vec{a}_{tan} = [-4\cos t, -2\sin t, 0]/(\sin^2 t + 4\cos^2 t)$

Note $\vec{a}_{tan}\cdot\vec{a}_{norm} = 0$ so perpendicular

$\frac{d^2s}{dt^2} = \frac{-3\cos t\sin t}{\sqrt{\sin^2 t + 4\cos^2 t}}$; $\vec{u}(t) = [-\sin t, 2\cos t, 0]/\sqrt{\sin^2 t + 4\cos^2 t}$

So $\frac{d^2s}{dt^2}\vec{u}(t)$ also gives $\vec{a}_{tan}$

$\vec{u}\,'(t) = [-4\cos t, -2\sin t, 0]/(\sin^2 + 4\cos^2 t)^{3/2}$

so $\vec{u}\,'(t)\frac{ds}{dt}$ also gives $\vec{a}_{norm}$

9.5#48 By example 6 on page 394

$s = Kt$ where $K = \sqrt{a^2+c^2}$

so $\vec{r}(s) = [a\cos(s/K), a\sin(s/K), cs/K]$

$k(s) = |\vec{u}\,'(s)| = |r''(s)| = [-(a/K^2)\cos(s/K), -(a/K^2)\sin(s/K), 0] = a/K^2$

we skipped torsion

NOTE: $\vec{r}\,'(t) = [-a\sin t, a\cos t, c]$; $\vec{r}\,''(t) = [-a\cos t, -a\sin t, 0]$

$\vec{r}\,'\cdot\vec{r}\,' = a^2 + c^2$; $\vec{r}\,''\cdot\vec{r}\,'' = a^2$; $\vec{r}\,'\cdot\vec{r}\,'' = 0$

and $\sqrt{(a^2+c^2)a^2 - 0}/(a^2+c^2)^{3/2} = a/(a^2+c^2) = a/K^2$

9.5#38 $\vec{r}(t) = 3.85\cdot10^8[\cos\frac{2\pi t}{2.36\cdot10^6}, \sin\frac{2\pi t}{2.36\cdot10^6}]$

$\therefore \vec{a}(t) = \vec{r}\,''(t) = 3.85\cdot10^8\left(\frac{2\pi}{2.36\cdot10^6}\right)^2\left[-\cos\frac{2\pi t}{2.36\cdot10^6}, -\sin\frac{2\pi t}{2.36\cdot10^6}\right]$

and $|\vec{a}(t)| = 3.85\cdot10^8\left(\frac{2\pi}{2.36\cdot10^6}\right)^2 = .00273$ meter/(second)^2