313 Homework 10

14.2

#8: $\frac{3}{4}$ is inside of C, so $\frac{1}{4}\oint_C \frac{dz}{(z-3/4)} = \frac{1}{4}2\pi i = \boxed{\frac{\pi i}{2}}$

#22: 0 and 2 are inside C, $\frac{7z-6}{z(z-2)} = \frac{3}{z}+\frac{4}{z-2}$

so $\oint_C \left[\frac{3}{z}+\frac{4}{z-2}\right]dz = 3\cdot 2\pi i + 4\cdot 2\pi i = \boxed{14\pi i}$

#24: 0 is not in region bdd by C, so $\boxed{0}$

#26: pts on real axis where $x \le -2$ are outside C, so $\boxed{0}$

14.3

#2: $\pm 2i$ are outside C, so $\boxed{0}$

#6: $\frac{1}{3}$ is inside C, so $\frac{1}{3}\oint_C \frac{e^{3z}\,dz}{z-1/3} = \frac{2\pi i}{3}e^{i} = \boxed{\frac{2\pi i}{3}(\cos 1 + i\sin 1)}$

#8: 1 is inside, -1 is outside, so $\oint_C \frac{dz}{(z-1)(z+1)} = 2\pi i \frac{1}{(z+1)}\Big|_1 = \boxed{\pi i}$

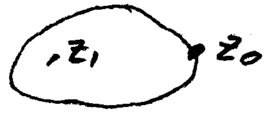
OR $\oint \frac{1}{2}\left(\frac{1}{z-1} - \frac{1}{2}\frac{1}{z+1}\right)dz = \frac{1}{2}(2\pi i + 0) = \pi i$

#16: 0 is outside region, $2i$ is inside so $\oint \frac{\sin z}{z}/(z-2i)\,dz$

$= 2\pi i \frac{\sin z}{z}\Big|_{2i} = \frac{2\pi i}{2i}\sin 2i = \frac{\pi}{2i}(e^{-2}-e^{2}) = \boxed{\pi i(e^{2}-e^{-2})/2}$

#18: For $z_1 \neq z_2$, $\oint_C\left[\frac{1}{z_2-z_1}\frac{1}{z-z_1} + \frac{1}{z_1-z_2}\frac{1}{z-z_2}\right]dz = \frac{2\pi i}{z_2-z_1} + \frac{2\pi i}{z_1-z_2} = 0$

For $z_1 = z_2$ $\oint_C \frac{dz}{(z-z_1)^2} = \frac{-1}{(z-z_1)}\Big|_{z_0}^{z_0} = \frac{-1}{(z_0-z_1)} + \frac{1}{(z_0-z_1)} = 0$



14.4

#4: 0 is inside C, so $\frac{2\pi i}{(2n)!}f^{(2n)}(0) = \frac{2\pi i}{(2n)!}(-1)^n\cos 0 = \boxed{\frac{2\pi i}{(2n)!}(-1)^n}$

#8: If a is inside $\boxed{\frac{2\pi i}{(n-1)!}e^{a}}$. If a is outside, $\boxed{0.}$

#10: 4 is inside $|z-3| = \frac{3}{2}$, so outside region so $\boxed{0}$

#12: 0 is outside region, but $2i$ is inside

So $\oint_C \frac{e^{2z}}{z}\frac{dz}{(z-2i)^2} = 2\pi i \frac{d}{dz}\frac{e^{2z}}{z}\Big|_{2i} = 2\pi i\left[-e^{2z}/z^2 + 2e^{2z}/z\right]_{2i}$

$= 2\pi i\, e^{4i}\left[\frac{1}{4} - i\right] = \boxed{\pi(\cos 4 + i\sin 4)\left(2 + \frac{i}{2}\right)}$