Sample problems



13.6 #19 $\arccos(2i) = -i \ln[2i + \sqrt{-4-1}] =$

$-i \begin{cases} \ln(\sqrt{5}+2) + (\frac{\pi}{2}+2k\pi)i \\ \ln(\sqrt{2}-2 + (-\frac{\pi}{2}+2k\pi)i \end{cases} = \begin{cases} (\frac{\pi}{2}+2k\pi) - i\ln(\sqrt{5}+2) \\ (-\frac{\pi}{2}+2k\pi) - i\ln(\sqrt{5}-2) \end{cases}$

Note that $(\sqrt{5}-2) = 1/(\sqrt{5}+2)$, so $\ln(\sqrt{5}-2) = -\ln(\sqrt{5}+2)$

So first set can be written as $(\frac{\pi}{2}+n\pi) - i(-1)^n \ln(\sqrt{5}+2)$, $n$ even

and " " " " " " " $(\frac{\pi}{2}+n\pi) - i(-1)^n \ln(\sqrt{5}+2)$, $n$ odd

So all together get $(\frac{\pi}{2}+n\pi) + (-1)^{n+1} i \ln(\sqrt{5}+2)$ OR

$2i = \cos x \cosh y - i \sin x \sinh y$. So $x = \frac{\pi}{2}+n\pi$, $\sin x = (-1)^n$

$\therefore 2 = -(-1)^n \frac{1}{2}(e^y - e^{-y})$ so $e^{2y} + 4(-1)^n e^y - 1 = 0$

$\therefore e^y = (-4(-1)^n \pm \sqrt{20})/2 = 2(-1)^{n+1} + \sqrt{5}$ since $2(-1)^n - \sqrt{5} < 0$

Hence $y = \ln(\sqrt{5} + 2(-1)^{n+1}) = (-1)^{n+1} \ln(\sqrt{5}+2)$

Hence $z = (\frac{\pi}{2}+n\pi) + (-1)^{n+1} i \ln(\sqrt{5}+2)$ as before

14.1 #19 $z(t) = t(1+i)$, $0 \le t \le 1$ $z'(t) = (1+i)$

$\int_C \operatorname{Re} z\, dz = \int_0^1 t(1+i)\,dt = \boxed{\frac{1}{2}(1+i)}$

#22 $\frac{d}{dz}(\cos z) = -\sin z$, so $\int_C \sin z\, dz = -\cos z \Big|_0^{2i}$

$= -\cos 2i + \cos 0 = 1 - \frac{1}{2}(e^{-2}+e^2) = \boxed{1 - \cosh 2}$

#26 $z(x) = x + ix^2$, $-1 \le x \le 1$ ; $z'(x) = 1 + i2x$

$\int_C \bar{z}\, dz = \int_{-1}^1 (x - ix^2)(1+2ix)\,dx = \int_{-1}^1 [x + 2x^3 + ix^2]\,dx = \boxed{\frac{2}{3}i}$

13.6 #8 $\sin(1+i) = \frac{1}{2i}[e^{i-1} - e^{1-i}] = \frac{1}{2i}[e^{-1}(\cos 1 + i \sin 1) - e(\cos 1 - i\sin 1)]$

$= \frac{1}{2}\sin 1\,(e^{-1}+e) + \cos 1\,\frac{1}{2}(e - e^{-1}) = \boxed{\sin 1 \cosh 1 + \cos 1 \sinh 1}$

OR plug directly into (6b) on page 627