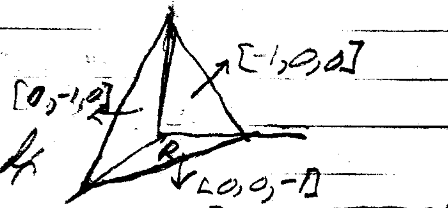
10.9:

#16 $\nabla\times\vec F=\vec 0$ so integral around <u>any</u> closed path $\oint_C \vec F\cdot d\vec r=0$

Notice $\nabla(x^3+y^3+z^3)/3=\vec F$ so $\vec F$ has a potential

10.7 #4 $\iiint_T e^{-x-y-z}\,dV=\iint_R\int_0^{2-x-z} e^{-x-y-z}\,dz\,dA$



$=\iint_R(e^{-x-y}-e^{-2})\,dA=\int_0^2\int_0^{2-x}(e^{-x-y}-e^{-2})\,dy\,dx$

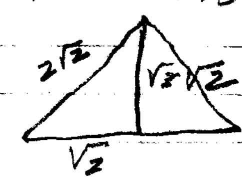
$=\int_0^2(e^{-x}-e^{-2}-e^{-2}(2-x))\,dx=[-e^{-x}-3e^{-2}x+e^{-2}x^2/2]_0^2=\boxed{1-5e^{-2}}$

OR Let $\vec F=[-e^{-x-y-z},0,0]$ Then $\iiint_T\nabla\cdot\vec F\,dV=\iint_S\vec F\cdot\vec n\,dS$

$=\int_0^2\int_0^{2-z}e^{-y-z}\,dy\,dz-\iint_S e^{-2}\frac{1}{\sqrt3}\,dS=\int_0^2(e^{-z}-e^{-2})\,dz-e^{-2}\frac{1}{\sqrt3}\cdot 2\sqrt3$

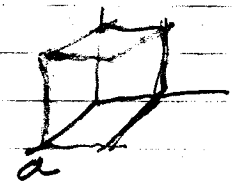
$=1-e^{-2}-2e^{-2}-2e^{-2}$ since surface

$=1-5e^{-2}$ area is $2\sqrt3$



#10 $I_x=\iiint(y^2+z^2)\,dV=\iint_R\int_0^a(y^2+z^2)\,dx\,dA=$

$\int_{-\frac c2}^{\frac c2}\int_{-\frac b2}^{\frac b2}a(y^2+z^2)\,dy\,dz=\boxed{\frac{1}{12}a(b^3c+c^3b)}$



#23 $\iiint_T(8x+2y+2z)\,dV=\iint_R\int_0^{2-x-y}12z\,dz\,dA=$

$6\int_0^2\int_0^{2-x}(2-x-y)^2\,dy\,dx=2\int_0^2(2-x)^3\,dx=2\frac{2^4}{4}=\boxed{8}$

OR $\iint_S\vec F\cdot\vec n\,dS$; $\int_0^2\int_0^{2-x}(-x)\,dz\,dx=\int_0^2(x^2-2x)\,dx=\frac83-4=-\frac43$

$\int_0^2\int_0^{2-x}-x^2\,dy\,dx=\int_0^2(x^3-2x^2)\,dx=4-\frac{16}{3}=-\frac43$

$\iint_S x^2\cos\alpha\,dS=\int_0^2\int_0^{2-y}(2-y-z)^2\,dz\,dy=\int_0^2\frac13(2-y)^3\,dy=\frac43$

$\iint_S x\cos\beta\,dS=\int_0^2\int_0^{2-x}x\,dz\,dx=\int_0^2(2x-x^2)\,dx=4/3$

So $7\cdot\frac43+\frac43-\frac43-\frac43=\boxed{8}$

10.8

#1 $\nabla^2 f = 4+4-8=0$ so harmonic $\nabla f = [4x, 4y, -8z]$

so for top $\iint_S \frac{\partial f}{\partial n} dS = \iint_S -8z\, dS = -8\iint_S dS = -8$

Other 5 faces gives $4, 4, 0, 0, 0$ so sum is $\boxed{0}$

#3 $\nabla f = [0, 6y, 0]$; $\nabla g = [2x, 0, 0]$; $\nabla^2 g = 2$

so L.H.S $= \iiint_T 6y^2 dV = \iint_R \int_0^1 6y^2\, dy\, dA = \iint_R 2\, dA = \boxed{2}$

for front face $\iint_S f \frac{\partial g}{\partial n} dS = \iint_S 3y^2 dS = \int_0^1\int_0^1 3y^2\, dy\, dz = 2$

integrals over other faces are 0 is $\boxed{2}$

10.6

#5 $\vec{N} = [-2u^2\cos v, -2u^2 \sin v, u]$ so

$\int_{-\pi}^{\pi}\int_0^4 (-2u^3\cos^2 v - 2u^3 \sin^2 v + u^3)\, du\, dv = \int_{-\pi}^{\pi}\int_0^4 (-u^3)\, du\, dv = \boxed{-128\pi}$

OR $\iint_S z\cos\gamma\, dS = \iint_R (x^2+y^2)\, dA = \int_0^{2\pi}\int_0^4 r^3\, dr\, d\theta = 2\pi \cdot 64$

$\iint_S x\cos\alpha\, dS = -2\iint_R \sqrt{z-y^2}\, dA = -2\int_{-4}^4 \int_{y^2}^{16} \sqrt{z-y^2}\, dz\, dy$

$= -2\,\frac{2}{3}\int_{-4}^4 (16-y^2)^{3/2} dy = -\frac{4}{3} 64 \cdot 4 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4\theta\, d\theta = -\frac{4}{3}\cdot 64\cdot 4 \cdot \frac{3}{8}\pi = -128\pi$

$\iint_S y\cos\beta\, dS = -128\pi$ so sum is -128π

#7 $\vec{N} = [-\cos u \sin^2 v, -\sin u \sin^2 v, -\cos u \sin v]$

$\iint_S \vec{F}\cdot\vec{N}\, dS = \int_{-\pi}^{\pi}\int_0^{2\pi} (-\cos u \sin^2 v - \sin u \sin^2 v - \cos u \sin v)\, du\, dv = 0$

OR $\iint_S 1\cos\gamma\, dS = \iint_R 1\, dA - \iint_R 1\, dA = 0$ as same for other 2 terms

#18 $z = \sqrt{1-x^2-y^2}$; $\vec{N} = [\frac{x}{z}, \frac{y}{z}, 1]$ so $|\vec{N}| = \frac{1}{z}$

$\iint_S G\, dS = \iint_R (ax+by+cz)\frac{1}{z}\, dA = 0 + 2b\int_0^1\int_0^{\pi} \frac{r\sin\theta}{\sqrt{1-r^2}}\, r\, d\theta\, dr + c\frac{\pi}{2}$

$= 2b\int_0^{2\pi} \frac{\sin^2 u}{\cos u}\cos u\, du + \frac{c\pi}{2} = 2b\frac{\pi}{4} + \frac{c\pi}{2} = \frac{\pi}{2}(b+c)$