

Math 110

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Exam # 2

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Clearly show your answers and how you got them. Follow the Honor Code.

- (3) 1. You roll 3 fair dice. Find the probabilities that:  
 a) All are 4's;    b) none are 4's.

$$a) \left(\frac{1}{6}\right)^3$$

$$b) \left(\frac{5}{6}\right)^3$$

- (3) 2.  $\frac{3}{4}$  of boxes contain a ball and  $\frac{1}{4}$  contain a doll. None contain both. You choose 2 boxes. Find the probability that you get a ball and a doll.

prob of ball on first, doll in second is  $\frac{3}{4} \cdot \frac{1}{4}$   
 " " doll " " ball " " "  $\frac{1}{4} \cdot \frac{3}{4}$

$$\therefore \boxed{2 \frac{3}{4} \cdot \frac{1}{4}}$$

- (4) 3. You choose 3 people at random, without replacement, from a group of 8 women and 4 men. Find the probabilities that:  
 a) all are women;    b) exactly 2 are women.

$$a) \frac{8}{12} \cdot \frac{7}{11} \cdot \frac{6}{10} = \boxed{\frac{14}{55}}$$

$$b) n(S) = C(12, 3) = \frac{12 \cdot 11 \cdot 10}{3!} = 220$$

$$n(E) = C(8, 2) \cdot C(4, 1) = 28 \cdot 4 = 112$$

$$\therefore P(E) = \frac{112}{220} = \boxed{\frac{28}{55}}$$

- (4) 4. Ann and Bob shoot independently at a target. The probability that Ann will hit the target is .2 and for Bob .3. Find the probabilities that:  
 a) both hit the target;    b) at least one hits the target.

$$a) .2 \cdot .3 = \boxed{.06}$$

$$b) .2 + .3 - .06 = \boxed{.44} = 1 - .8 \cdot .7$$

since at least one hits unless both miss.

- (3) 5. 3 people are chosen at random. Find the probability that at least 2 were born on the same day of the week.

$1 - \frac{6}{7} \cdot \frac{5}{7} = \boxed{\frac{19}{49}}$  since at least 2 were born on same day unless all were born on different days

- (4) 6. 20% of tax returns are incorrect. 90% of incorrect returns are rejected. 95% of correct returns are accepted. Find the probabilities that:

a) a return is rejected, given that it is correct;

b) a return is correct, given that it is rejected.

C: correct  
I: incorrect  
R: rejected

Given:  $P(R|I) = .9$ ;  $P(R'|C) = .95$   
 $P(I) = .2$ ;  $P(C) = .8$

a)  $P(R|C) = 1 - P(R'|C) = 1 - .95 = \boxed{.05}$

b)  $P(R) = P(R|C)P(C) + P(R|I)P(I) = .05 \cdot .8 + .9 \cdot .2 = .22$

$P(C|R) = \frac{P(R|C)P(C)}{P(R)} = \frac{.05 \cdot .8}{.22} = \boxed{\frac{2}{11}}$

- (4) 7. You roll 2 fair dice.
- a) Draw a histogram for the probability distribution of the sum of the numbers on the 2 dice.
- b) Are the events A = "the sum is 5" and B = "the red die is 3": disjoint, i.e. mutually exclusive; independent? Explain.

R	2	3	4	5	6	7	8	9	10	11	12
$P(X=R)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

See figure 9 on page 351

b) Not disjoint because both can occur on the same trial, i.e. 3 on red and 2 on green.

c) Not independent because  $P(B) = \frac{1}{6} \neq \frac{1}{4} = P(B|A)$   
OR  $P(A) = \frac{4}{36} \neq \frac{1}{6} = P(A|B)$