

Math 110
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Exam # 1
February 18, 2004

Clearly show the calculations to solve the following. Follow the Honor Code.

- (2) 1. Let $U = \{a, b, c, d, e, f\}$, $R = \{b, c, f\}$, $S = \{a, b, c, e\}$.
List the elements which are not in both R and S .

$$(R \cap S)' = \{b, c\}' = \{a, d, e, f\}$$

- (3) 2. From a group of 45 students, 30 take art or biology, 15 take biology, and 22 take art. How many take:
a) both art and biology? b) neither art nor biology?

$$\begin{aligned} m(U) &= 45 & a) \quad 30 &= 15 + 22 - m(A \cap B) \\ m(A \cup B) &= 30 & \therefore m(A \cap B) &= 15 + 22 - 30 = \boxed{7} \\ m(B) &= 15 & b) \quad m(A \cup B)' &= m(U) - m(A \cup B) \\ m(A) &= 22 & &= 45 - 30 = \boxed{15} \end{aligned}$$

- (2) 3. There are 2 buses from Washington to New York, 3 trains from Washington to New York, and 4 planes from New York to Boston. In how many ways can you travel from Washington to Boston via New York?

There are $2 + 3$ ways from Wash to NY.

$$\therefore 5 \cdot 4 = \boxed{20} \text{ ways from Wash to Boston}$$

- (2) 4. If you count the number of permutations of 12 objects taken 4 at a time, each combination will be counted how many times?

each is counted $4!$ ways since a combination of 4 can be put in order in $4 \cdot 3 \cdot 2 \cdot 1$ ways

- (2) 5. In how many ways can 50 people fill 12 different jobs?

Choose 12 from 50, no repeat, order counts

$$P(50, 12) = 50 \cdot 49 \cdot 48 \cdot \dots \cdot 39$$

- (4) 6. a) How many sequences of 0's and 1's are there? of length 10
 b) In how many of these will there be exactly 3 0's?
 c) In how many will there be at least 3 0's?

a) 2 choices for each, so 2^{10}

b) choose 3 places for the 0's, so $C(10, 3) = \frac{10 \cdot 9 \cdot 8}{3!}$

c) Can't have 0, 1, or 2 0's, so $2^{10} - 1 - 10 - \frac{10 \cdot 9}{2}$

- (8) 7. For a group of 10 women and 10 men, in how many ways can you:
 a) choose 2 people of the opposite sex?
 b) choose 2 people of the same sex?
 c) form 10 pairs?
 d) form 10 pairs, each containing a man and a woman?

a) choose 1 of 10 women and 1 of 10 men, so $10 \cdot 10 = 100$

b) choose 2 of 10 women or 2 of 10 men, so

$$\frac{10 \cdot 9}{2} + \frac{10 \cdot 9}{2} = 90 \quad \text{OR} \quad \frac{20 \cdot 19}{2} - 100 = 90$$

c) unordered partition $\frac{20!}{2^{10} \cdot 10!}$

d) Choose a different man to watch with each woman,
 so $P(10, 10) = 10!$

- (2) 8. In how many ways can you choose at least 10 people from a group of 12?

$$C(12, 10) + C(12, 11) + C(12, 12) = \frac{12 \cdot 11}{2} + 12 + 1$$