Math 110
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Exam # 2
October 27, 2008

Clearly show your answers and how you got them. Follow the Honor Code.

(1) 1. What does it mean to say that the probability of rain in the county tomorrow is .5:
a) it will rain for 12 hours tomorrow;
b) it will rain on 5 of next 10 days;
c) it rains on half the days with similar conditions;
d) it will rain in half the county tomorrow.

c) because prob. gives relative frequency

(2) 2. You roll 2 fair dice. Are "the sum is 2" and "the sum is 3" equally likely? Explain.

NO because to get 2, you must get 1 on both dice; to get 3, you get 1 and 2 in either order

(2) 3. You choose 2 people from a group of 4 women and 8 men. Are "the first is male" and "the second is male" a) disjoint?; b) independent? Explain.

a) NO, both can be male

b) NO, if first is male, then prob. that second is male is decreased

(3) 4. For the previous problem, find the probabilities that:
a) both are male; b) at least one is male; c) the 2 are the same sex.

A is 1st is male; B is second is male

a) $P(A \cap B) = \frac{8}{12} \cdot \frac{7}{11} = \frac{14}{33}$

b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{8}{12} + \frac{8}{12} - \frac{8}{12} \cdot \frac{7}{11}$
$= 1 - P(A' \cap B') = 1 - \frac{4}{12} \cdot \frac{3}{11} = \frac{10}{11}$

c) $P(A \cap B) + P(A' \cap B') = \frac{8}{12} \cdot \frac{7}{11} + \frac{4}{12} \cdot \frac{3}{11} = \frac{17}{33}$

(2) 5. Which has a) the largest probability; b) the smallest probability? Explain.
i) a man will kill his wife;
ii) a man has beaten his wife, given that he killed her;
iii) a man will kill his wife, given that he has beaten her.

a) ii since most men who kill their wives have beaten them first

b) i since having a bet in his wife makes it more likely that he will kill her

(3) 6. You make 6 letter word from the letters w,x,y,z. Find the probabilities that there is:
a) at least one z; b) exactly one z.

S: ways to choose 6 from 4; rep: YES; order YES

$n(S) = 4^6$

a) $n(E') = 3^6$, so $P(E) = \boxed{1 - \frac{3^6}{4^6}}$

b) 6 ways to put z; 3^5 ways to choose the rest, so $P(E) = \boxed{(6 \cdot 3^5)/4^6}$

(3) 7. A group consists of 6 women and 4 men. If you choose 8 people, what are the probabilities that you will choose: a) all the women; b) none of the men?

S: choose 8 from 10; rep: NO; order: N/A; $n(S) = C(10,8) = \frac{10 \cdot 9}{2}$

a) choose 6 of 6 women and 2 of 4 men; $n(E) = C(6,6) \cdot C(4,2)$

$= 1 \cdot \frac{4 \cdot 3}{2}$ so $P(E) = \boxed{6/45}$

b) $\boxed{0}$ since it is impossible to choose all 8 from 6 women

(3) 8. 10% of a population are liars. 80% of them fail a polygraph test.
95% of the truth tellers pass the test. What are the probabilities that:
a) a person fails the test, given that he is a truth teller;
b) a person is a truth teller, given that he fails the test.

L: liar
F: fail
$P(L) = .1$; $P(L') = .9$
$P(F|L) = .8$
$P(F'|L') = .95$

a) $P(F|L') = 1 - .95 = \boxed{.05}$

b) $P(F) = .1 \cdot .8 + .9 \cdot .05 = .125$

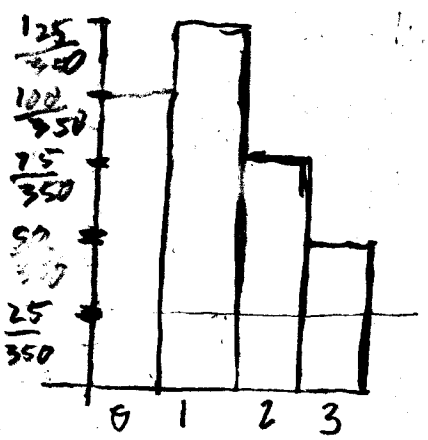
$P(L'|F)\,P(F) = P(F|L') \cdot P(L') = .05 \cdot .9$

$\therefore P(L'|F) = (.05 \cdot .9)/(.1 \cdot .8 + .9 \cdot .05)$

$= \boxed{.045/.125}$

(3) 9. 100 insureds have 0 accidents, 125 have 1, 75 have 2, and 50 have 3. total 350
a) Draw a histogram for the relative frequencies of accidents per insured.
b) What are the probabilities that an insured will have at least 2 accidents; at most 2?

X	rel freq
0	100/350
1	125/350
2	75/350
3	50/350



$P(X \geq 2) = \frac{75}{350} + \frac{50}{350} = \boxed{\frac{125}{350}}$

$P(X \leq 2) = 1 - \frac{50}{350} = \boxed{\frac{300}{350}}$