Math 110
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Exam # 1
September 26, 2008

Clearly show the calculations to solve the following. Except for the first problem, it is not necessary to actually calculate the answer. Follow the Honor Code.

(2) 1. How many ways can you choose 99 people from a group of 100?

$C(100,99) = C(100,1) = \boxed{100}$

(3) 2. From a group of 500 people, 350 play tennis, 250 play hockey, 200 play both. How many a) play tennis, but not hockey; b) play neither.
c) If you choose somebody at random, what is the probability that he plays at least one of the sports?

$n(U) = 500$
$n(T) = 350$
$n(H) = 250$
$n(T \cap H) = 200$

a) $n(T \cap H') = n(T) - n(T \cap H)$
$= 350 - 200 = \boxed{150}$

b) $n(T \cup H) = n(T) + n(H) - n(T \cap H)$
$= 350 + 250 - 200 = 400$

~~So~~ $n(T' \cap H') = 500 - 400 = \boxed{100}$

c) $P(T \cup H) = n(T \cup H)/n(U) = \frac{400}{500} = \boxed{\frac{4}{5}}$

(2) 3. If you have 6 sandwiches, 3 soups, and 4 salads to choose from, in how many ways can you choose a sandwich and either a soup or a salad?

Choose soup or salad in $4 + 3 = 7$ ways

So have $\boxed{6 \cdot 7}$ choices

(2) 4. How many 12 letter words have 4 R's, 3 S's, and 5 T's?

Choose where to place 4 R's in $C(12,4)$ ways
" " " " 3 S's in $C(8,3)$ "
Place 5 T's in remaining places

So $C(12,4) \cdot C(8,3) = \boxed{\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{4! \cdot 3!}}$

(4) 5. How many 6 letter words can be made from w,x,y,z, if there is:
a) exactly 1 z; b) at most 1 z; c) at least 1 z?

a) choose where to put z in 6 ways. Fill other 5 places in 3^5 ways
so $\boxed{6 \cdot 3^5}$

b) If no z's, have 3^6 ways. So $\boxed{3^6 + 6 \cdot 3^5}$

c) 4^6 ways altogether. So $\boxed{4^6 - 3^6}$

(3) 6. A salad bar offers 8 ingredients (including lettuce) and 4 dressings. How many different salads that contain at least lettuce and at most 1 dressing are possible?

choose other ingredients in 2^7 ways
choose dressing in $1+4=5$ ways
So $\boxed{2^7 \cdot 5}$

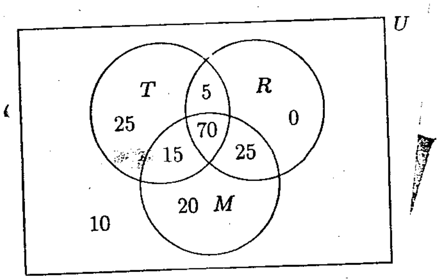
(3) 7. In how many ways can 4 men and 4 women be seated:
a) in a row; b) in a row with nobody next to someone of the same sex?
c) What is the probability that nobody is seated next to someone of the same sex?

a) $P(8,8) = \boxed{8!}$ since we are permuting 8 people.

b) $\boxed{8 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1}$ since 8 choices for first, 4 for second, etc.

c) $\dfrac{8 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2}{8!} = \dfrac{P(E)}{P(S)}$

(3) 8. For the sets shown in this diagram, how many are:
a) in R and in T, but not in M; b) in exactly 1 of R,T,M;
c) at most 1 of R,T,M?



a) 5

b) $25+20+0$

c) $25+20+0+10$