Math 108
W.T.Kiley

Exam 1
October 8, 2009

Clearly show your work. Follow the Honor Code.

(4) 1. A manufacturer has overhead of \$3,000 and production costs of \$20 per item. $q = (300 - 2p)$ items can be sold at a price of \$p per item. Find:
a) the total cost for a production of 100 items;
b) the total revenue if the price is \$50 per item;
c) the profit if the price is \$50 per item.

a) $C(q) = 3000 + 20q$; $C(100) = 3000 + 20 \cdot 100 = \boxed{5,000}$

b) $R(p) = q \cdot p = (300 - 2p)p$

$R(50) = (300 - 2 \cdot 50)50 = 200 \cdot 50 = \boxed{10,000}$

c) $C(p) = 3000 + 20(300 - 2p)$; $C(50) = 3000 + 20 \cdot 200 = 7000$

$P(50) = R(50) - C(50) = 10,000 - 7,000 = \boxed{3000}$

(8) 2. $y = f(x) = -x^2 + 2x + 3$.
a) Find the largest and smallest values of y, if they exist.
b) Find the derivative of f.
c) Find where f is increasing and where it is decreasing
d) Find the equation of the tangent line to the graph at the point where x = 2.
e) Sketch the graphs of y = f(x) and the tangent line found above.

a) vertex at $x = \frac{-2}{-2} = 1$; $f(1) = -1 + 2 + 3 = 4$

coeff of x^2 is neg., so $\cap$ $\therefore$ No smallest; largest is $\boxed{4}$

b) $f'(x) = -2x + 2$

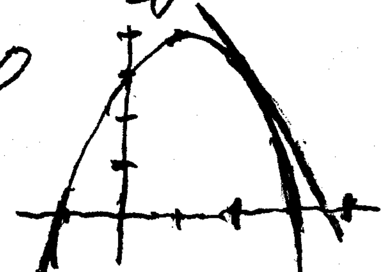
c) $f'(x) = 2(1-x) > 0$ for $x < 1$, so f is increasing on $(-\infty, 1)$

$f'(x) = 2(1-x) < 0$ for $x > 1$, so f is decreasing on $(1, +\infty)$

d) $f'(2) = -4 + 2 = 2$; $f(2) = -4 + 4 + 3 = 3$

$y = -2x + b$; $3 = -4 + b$; $b = 7$ so $\boxed{y = -2x + 7}$

e) intercepts at $(0,3)$; $(-1,0)$, $(3,0$



(4) 3. f(x) = x^2 + 2 if x < 2 and f(x) = 3x + 1 if 2 < x. Find, if they exist:
a) f(2); b) $\lim_{x \to 2}$ f(x); c) f'(2). d) Is f continuous at 2? Explain.

a) neither rule applies to $x=2$, so $f(2)$ DNE

b) $\lim_{x\to 2^-} f(x) = 2^2+2 = 6$; $\lim_{x\to 2^+} f(x) = 3\cdot 2+1 = 7$

$6 \neq 7$, so $\lim_{x\to 2} f(x)$ DNE

c) since $f(2)$ DNE, $f'(2)$ is not defined either

d) f is not continuous at 2 because $f(2)$ DNE

(6) 4. f(x) = (2x^2 + 3x)/(x^2 – x). Find, either finite or infinite:
a) $\lim_{x\to 0}$ f(x); b) $\lim_{x\to 1^+}$ f(x); c) $\lim_{x\to +\infty}$ f(x).

a) $\lim_{x\to 0} \frac{x(2x+3)}{x(x-1)} = \lim_{x\to 0} \frac{2x+3}{x-1} = \frac{3}{-1} = \boxed{-3}$

b) $\lim_{x\to 1^+} \frac{2x^2+3x}{x^2-x} = \boxed{+\infty}$ because $2x^2+3x \to 5$ and $x^2-x \to 0^+$

c) $\lim_{x\to +\infty} \frac{2x^2+3x}{x^2-x} = \lim_{x\to +\infty} \frac{2+3/x}{1-1/x} = \boxed{2}$

(4) 5. Find the derivative using the DEFINITION of f(x) = 5 – x^2.

$$\lim_{h \to 0} \frac{5-(x+h)^2-(5-x^2)}{h} =$$

$$\lim_{h \to 0} \frac{5-x^2-2xh-h^2-5+x^2}{h} =$$

$$\lim_{h \to 0} \frac{-2xh-h^2}{h} =$$

$$\lim_{h \to 0} (-2x-h) = \boxed{-2x}$$

(4) 6. Find the derivatives of:

a) (x^2 + 1)/(2 + 3x); b) $\sqrt{3x\text{\^{}}2 + 7}$.

a) $$\frac{d}{dx}(x^2+1)(2+3x)^{-1} = 2x(2+3x)^{-1}-(x^2+1)(2+3x)^{-2}3$$

$$= \frac{2x(2+3x)-(x^2+1)3}{(2+3x)^2} = \frac{3x^2+4x-3}{(2+3x)^2}$$

b) $$\frac{d}{dx}(3x^2+7)^{1/2} = \frac{1}{2}(3x^2+7)^{-1/2}6x$$

$$= \frac{3x}{\sqrt{3x^2+7}}$$