Math 108
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Exam 2
April 16, 2009

Clearly show your work. Follow the Honor Code. Write your name on the back, last name first.

(4) 1. Find the derivatives of: a) f(x) = e^(x^2); b) f(x) = ln(2 + 3x).

a) $f(x) = e^u$ where $u = x^2$; $\frac{d}{du} e^u = e^u$

so $f'(x) = \boxed{e^{(x^2)}(2x)}$

b) $f(x) = \ln u$ where $u = 2+3x$; $\frac{d}{du} \ln u = \frac{1}{u}$

so $f'(x) = \boxed{\left(\frac{1}{2+3x}\right)3}$

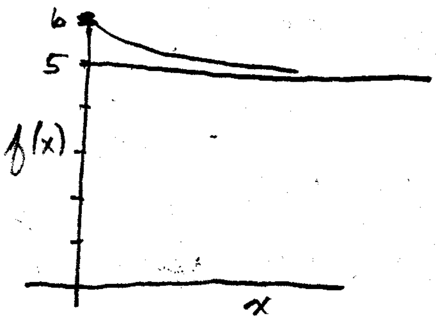
(4) 2. Sketch the graph of f(x) = 5 + e^(-x) where 0≤ x.

$f(0) = 5+1=6$; $f(x) > 5$

$\lim_{x \to +\infty} f(x) = 5$

$f'(x) = -e^{-x} < 0$, so $f \searrow$

$f''(x) = e^{-x} > 0$, so $f \cup$



(4) 3. You can sell 100 – 2p widgets at a price of $p per widget. It costs you $20 per widget.
a) How much profit do you make if you sell widgets for $40 each?
b) What price should you charge in order to produce the largest profit?

$R(p) = (100-2p)p$; $C(p) = (100-2p)20$; $P(p) = R(p) - C(p) = (100-2p)(p-20)$

a) $P(40) = 20 \cdot 20 = \boxed{400}$

b) $P'(p) = -4p + 140$; $P'(p) = 0$ for $p = 35$; $P'' = -4 < 0$, so P is $\cap$

$\therefore$ $\boxed{p = 35}$ gives the absolute max

Note $P(35) = 30 \cdot 15 = 450$. Reduce the price

(4) 4. You invest at 5% annual interest compounded continuously.
a) How much must you invest in order to have $20,000 in 10 years?
b) How long must you wait in order for $10,000 to grow to $20,000?

$B = P e^{.05t}$

a) $B = 20{,}000$, $t = 10$. So $20{,}000 = Pe^{.5}$, $P = \frac{20{,}000}{e^{.5}} = \boxed{12{,}130.61}$

b) $B = 20{,}000$, $P = 10{,}000$. So $20{,}000 = 10{,}000\, e^{.05t}$

$2 = e^{.05t}$; $\ln 2 = .05t$; $t = \frac{\ln 2}{.05} = \boxed{13.86}$

(10) 5. f(x) = (ln x)/x. Find: a) f '(x); b) f "(x); c) where f is increasing / decreasing; d) where f is concave up / down; e) coordinates of relative extrema; f) coordinates of inflection points;.

a) $f'(x) = \frac{1}{x}\frac{1}{x} - \ln x \frac{1}{x^2} = \boxed{\frac{1-\ln x}{x^2}}$

b) $f''(x) = -\frac{1}{x}\frac{1}{x^2} - (1-\ln x)(-\frac{2}{x^3}) = \boxed{(2\ln x - 3)/x^3}$

c) $f'(x) = 0$ for $1 = \ln x$, i.e. $x = e$

$f'(x) > 0$ for $0 < x < e$, so $f \nearrow$

$f'(x) < 0$ for $e < x$, so $f \searrow$

d) $f''(x) = 0$ for $\ln x = \frac{3}{2}$, i.e. $x = e^{1.5}$

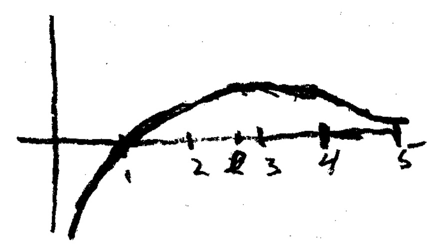
$f''(x) < 0$ for $0 < x < e^{1.5}$, so $f \cap$

$f''(x) > 0$ for $e^{1.5} < x$, so $f \cup$

e) rel max is $f(e) = \frac{1}{e}$, so $(e, \frac{1}{e}) = \boxed{(2.72, .368)}$

f) inf pt is $f(e^{1.5}) = 1.5e^{-1.5}$

so $(e^{1.5}, 1.5e^{-1.5}) = \boxed{(4.48, .335)}$



(4) 6. g(1) = 2; g'(x) < 0 for 0 < x < 1; g'(x) > 0 for 1 < x; g"(x) > 0 for 0 < x < 3; g"(x) < 0 for 3 < x; $\lim_{x\to 0^+} g(x) = +\infty$; $\lim_{x\to +\infty} g(x) = 4$. Sketch the graph of g.

	(0, 1)	(1, 3)	(3, +∞
g'	−	+	+
g	↘	↗	↗
g"	+	+	−
g	∪	∪	∩