Math 108
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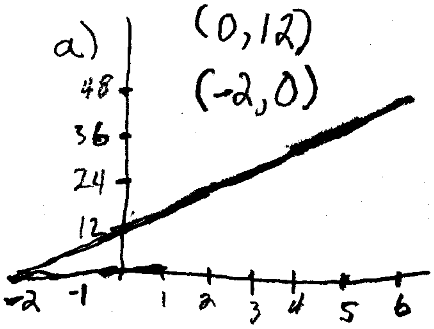
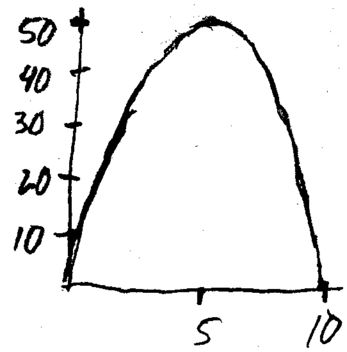
Clearly show your work. Follow the Honor Code.

Exam 1
February 26, 2009

(3) 1. Sketch the graphs of the following. (Show your work) .
a) y = 12 + 6x; b) y = 20x – 2x^2.





intercepts (0,0), (10,0)

vertex $x = \frac{-20}{-4} = 5$

$y = 100 - 50 = 50$

(4) 2. Find the derivatives, if they exist, of the functions above at x = 8.

a) $y' = 6$

$\boxed{y'(8) = 6}$

b) $y' = 20 - 4x$

$\boxed{y'(8) = 20 - 32 = -12}$

(4) 3. Find the equations of the tangent lines for the functions above at the points where x = 8.

a) $y(8) = 12 + 48 = 60$

$y = 6x + b$

$60 = 6 \cdot 8 + b$

$12 = b$

$\boxed{y = 6x + 12}$

b) $y(8) = 160 - 128 = 32$

$y = -12x + b$

$32 = -12 \cdot 8 + b$

$= 128$

$\boxed{y = -12x + 128}$

(7) 4. x cases of widgets can be sold for a price of p = 20 - 2x \$ per case. It costs \$12 for rent and \$6 per case for materi
Find: a) The revenue that results from producing 8 cases of widgets;
b) The rate at which revenue is changing with respect to production when production is 8 cases;
c) The production that produces the maximum revenue;
d) The total cost of producing 8 cases of widgets;
e) The break even productions (zero profit.)

a) $R = px = (20-2x)x = 20x - 2x^2 \therefore R(8) = \boxed{32}$

b) $R' = 20 - 4x \therefore R'(8) = \boxed{-12}$

c) As seen in graph 1b, $R(x)$ is max when $\boxed{x = 5}$

d) $C = 12 + 6x \therefore C(8) = 12 + 48 = \boxed{60}$

e) $P = R - C = 20x - 2x^2 - 12 - 6x = -2(x^2 - 7x + 6) = -2(x-1)(x-6)$

so $\boxed{x = 1 \text{ or } 6}$ These are the points where the graphs in 1a and 1b intersect.

(6) 5. f(x) = 3x – 1 for x < 2, f(x) = x^2 + 1 for x ≥ 2. Find, if they exist:
a) f(2); b) $\lim_{x\to 2}$ f(x); c) $\lim_{h\to 0}$ [f(2 + h) – f(2)]/h; d) f'(2).

a) $f(2) = 2^2 + 1 = \boxed{5}$; b) $\lim_{x\to 2^-} f(x) = 3\cdot 2 - 1 = 5$; $\lim_{x\to 2^+} f(x) = 2^2 + 1 = 5$; so $\lim_{x\to 2} f(x) = \boxed{5}$

c) $\lim_{h\to 0^+} \frac{(2+h)^2 + 1 - 5}{h} = \lim_{h\to 0^+} \frac{(4 + 4h + h^2) + 1 - 5}{h} = \lim_{h\to 0^+} \frac{4h + h^2}{h}$

$= \lim_{h\to 0^+} (4 + h) = 4$; $\lim_{h\to 0^-} \frac{3(2+h) - 1 - 5}{h} = \lim_{h\to 0^-} \frac{3h}{h} = 3 \neq 4$, so DNE

d) The [illegible] is the definition of $f'(2)$, so DNE

(6) 6. f(x) = (x^2 – 1)/(x^2 – x). Find, either finite or infinite:
a) $\lim_{x\to 2}$ f(x); b) $\lim_{x\to 1}$ f(x); c) $\lim_{x\to 0}$ f(x); d) $\lim_{x\to +\infty}$ f(x); e) f'(2).

a) $f(2) = \frac{2^2 - 1}{2^2 - 2} = \boxed{\frac{3}{2}}$; b) $\lim_{x\to 1} \frac{(x+1)(x-1)}{x(x-1)} = \lim_{x\to 1} \frac{(x+1)}{x} = \boxed{2}$; d) $\lim_{x\to\infty} (1 + \frac{1}{x}) = \boxed{1}$

since $\frac{1}{x} \to 0$

c) $\lim_{x\to 0^-} \frac{x+1}{x} = -\infty$ since $x + 1 > 0$ and $x < 0$;

$\lim_{x\to 0^+} \frac{x+1}{x} = +\infty$ since $x + 1 > 0$ and $x > 0$.

e) $f'(x) = \frac{(x^2 - x)(2x) - (x^2 - 1)(2x - 1)}{(x^2 - x)^2} \therefore f'(2) = \frac{2\cdot 4 - 3\cdot 3}{4} = \boxed{-\frac{1}{4}}$