Topology, Arithmetic, & Dynamics Seminar

The space of local overrings of an integral domain (preliminary report)

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Since the time of Grothendieck, algebraic geometry has been founded on the notion of covering an algebraic variety or scheme with affine schemes $\operatorname{Spec} A$. The A in question are commutative rings, and $\operatorname{Spec} A$ is to be understood as a topological space of prime ideals together with the associated structure sheaf that allows one to recover the ring A. Moreover, this is better than the previous approach of just looking at the *maximal* ideals of A, as said approach hides all the higher dimensional subschemes that live inside $\operatorname{Spec} A$. However, even Grothendieck's point of view hides a lot of information in the structure sheaf that is invisible on the level of topological spaces. In the current investigation, we assign a new topological space $\operatorname{Loc} A$ (even with a sheaf structure that makes it a locally ringed space) to an integral domain, thus attempting to provide a new foundation for algebraic geometry. The space $\operatorname{Loc} A$ contains $\operatorname{Spec} A$ as a dense retract. The Zariski-Riemann space of A in its fraction field is also a dense subset. Moreover, the topology of Loc is sensitive enough to detect where an algebraic curve is smooth (unlike the topology of Spec). Given a map of domains $A \to B$, we construct a map $\operatorname{Loc} B \to \operatorname{Loc} A$ that interacts well with the retraction comparison maps to Spec. In many cases of interest, this map is continuous. We do not know yet whether this map assignment is functorial in general (i.e. does it preserve composition?), but we have reduced this question to the case of local homomorphisms. This work is joint with Chris Manon and Jay Shapiro.

Date: **Friday, April 24, 2015**
Time: **2:30-3:30pm**
Place: **4106 Exploratory Hall**

For now, the seminar is BYOC^2. For special accommodations, please contact Sean Lawton via email at `slawton3@gmu.edu`.