In many different geometric settings, the vector bundles over a given space usually say a lot about the geometry of that space. For example, in the curve setting, where we have restricted the dimension of our space to be 1, we can get a hold of all sorts of interesting geometry by studying the moduli space of vector bundles. We can also restrict the rank of our vector bundles, say by studying bundles of rank 1 on projective varieties, where they keep track of ways to put coordinates on our space.

I’ll focus on the vector bundles over a more combinatorially restricted class of spaces from algebraic geometry, toric varieties. A toric variety comes equipped with a large group of symmetries, namely an effective action of an algebraic torus of the same dimension. It makes sense then to study the vector bundles on such a toric variety which also have an action by that same torus. I’ll explain Klyachko’s classification of these objects, along with some related work by Di Rocco, Jabbusch, and Smith. Then I’ll describe some recent joint work with Kiumars Kaveh where we give a different classification using piecewise linear geometry and spherical Bruhat-Tits buildings. This alternative classification admits an extension to a classification result for toric algebraic principal bundles for an arbitrary connected reductive group.