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Solution of Quiz 9

1 Find the absolute maximum and absolute minimum (if any) of the function $f(x) = x^3 + 3x^2 - 9x + 5$ on the interval $-2 \le x \le 2$.

Solution. From the derivative

$$f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x+3)(x-1)$$

we see that the critical numbers are $x = -3$ and $x = 1$. Of these, $x = 1$ lies in the interval $-2 < x < 2$. Compute $f(x)$ at the critical number $x = 1$ and at endpoints $x = -2$ and $x = 2$.

$$\begin{aligned} f(1) &= (1)^3 + 3(1)^2 - 9(1) + 5 = 0 \\ f(-2) &= (-2)^3 + 3(-2)^2 - 9(-2) + 5 = 27 \\ f(2) &= (2)^3 + 3(2)^2 - 9(2) + 5 = 7 \end{aligned}$$

Compare these values to conclude that the absolute maximum of $f(x)$ on the interval $-2 \le x \le 2$ is $f(-2) = 27$ and the absolute minimum is $f(1) = 0$.

2 A manufacturer estimates that when q units of a particular commodity are produced each month, the total cost will be $C(q) = q^2 - 10q + 25$ dollars and all q units can be sold at a price of $p(q) = 20 - q$ dollars per unit. At what level of production is the average cost per unit $A(q) = \dfrac{C(q)}{q}$ minimized?

Solution. The average cost is

$$A(q) = \frac{C(q)}{q} = \frac{q^2 - 10q + 25}{q} = q - 10 + 25q^{-1}$$

for $q > 0$. Since

$$A'(q) = 1 - 25q^{-2} = 1 - \frac{25}{q^2} = \frac{q^2 - 25}{q^2} = \frac{(q+5)(q-5)}{q^2}$$

the only critical number in the interval $q > 0$ is $q = 5$. Since

$$A''(q) = (-48)(-2)q^{-3} = \frac{96}{q^3} > 0 \quad \text{when} \quad q > 0$$

it follows from the second derivative test for absolute extrema that average cost $A(q)$ is minimized when $q = 5$.