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Solution of Quiz 6

1 Suppose the total cost in dollars of manufacturing x units is $C(x) = x^2 + 10x + 200$.

(a) Use marginal analysis to estimate the cost of manufacturing the 11th unit.

Solution. The marginal cost is $C'(x) = 2x + 10$.

The cost of producing the 11th unit is the change in cost as x increases from 10 to 11 and can be estimated by the marginal cost

$$C'(10) = 2 \cdot 10 + 10 = \$30$$

(b) What is the actual cost of manufacturing the 11th unit?

Solution. The actual cost of manufacturing the 11th unit is

$$\begin{aligned} C(11) - C(10) &= \left[11^2 + 10 \cdot 11 + 200\right] - \left[10^2 + 10 \cdot 10 + 200\right] \\ &= [121 + 110 + 200] - [100 + 100 + 200] \\ &= 431 - 400 = \$31 \end{aligned}$$

2 Find $\dfrac{dy}{dx}$ if $x^3 + y^3 = x^2 + 2y$.

Solution. Differentiating both sides of the given equation with respect to x, we get

$$\begin{aligned} \frac{d}{dx}\left[x^3 + y^3\right] &= \frac{d}{dx}\left[x^2 + 2y\right] \\ 3x^2 + 3y^2\frac{dy}{dx} &= 2x + 2\frac{dy}{dx} \\ (3y^2 - 2)\frac{dy}{dx} &= 2x - 3x^2 \\ \frac{dy}{dx} &= \frac{2x - 3x^2}{3y^2 - 2}. \end{aligned}$$