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Solution of Quiz 5

1 Find the derivative $f'(1)$ for the function $f(x) = \dfrac{x^2-1}{x^2+1}$.

Solution. By the quotient rule, we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[\frac{x^2-1}{x^2+1}\right] \\ &= \frac{(x^2+1)\frac{d}{dx}\left[x^2-1\right] - (x^2-1)\frac{d}{dx}\left[x^2+1\right]}{(x^2+1)^2} \\ &= \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2} \\ &= \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2} \\ &= \frac{4x}{(x^2+1)^2} \end{aligned}$$

Thus

$$f'(1) = \frac{4\cdot 1}{(1^2+1)^2} = 1.$$

2 Find the second derivative of the function $f(x) = x^6 - 2x^5 - 6x$.

Solution. The first derivative is

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[x^6 - 2x^5 - 6x\right] \\ &= 6x^5 - 2(5x^4) - 6 \\ &= 6x^5 - 10x^4 - 6 \end{aligned}$$

and then the second derivative is

$$\begin{aligned} f''(x) &= \frac{d}{dx}\left[f'(x)\right] \\ &= \frac{d}{dx}\left[6x^5 - 10x^4 - 6\right] \\ &= 6(5x^4) - 10(4x^3) \\ &= 30x^4 - 40x^3 \end{aligned}$$

3 Find the derivative of the function $f(x) = (x^2 - 2x + 2)^7$.

Solution. Apply the general power rule to get

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[(x^2-2x+2)^7\right] \\ &= 7(x^2-2x+2)^6\frac{d}{dx}(x^2-2x+2) \\ &= 7(x^2-2x+2)^6(2x-2) \\ &= 14(x^2-2x+2)^6(x-1) \end{aligned}$$