

## Solution of Quiz 4

**1** Find the equation of the tangent line to the curve  $y = \sqrt{x^3} + \frac{2}{x}$  at the point where  $x = 1$ .

**Solution.** when  $x = 1$ , the corresponding  $y$  coordinate on the given curve is  $y = \sqrt{1^3} + \frac{2}{1} = 3$ , so the point of tangency is  $(1, 3)$ .

The derivative of the given function with respect to  $x$  is

$$\frac{dy}{dx} = \frac{d}{dx} [x^{3/2} + 2x^{-1}] = \frac{d}{dx} [x^{3/2}] + 2\frac{d}{dx} [x^{-1}] = \frac{3}{2}x^{1/2} + 2(-1)x^{-2} = \frac{3}{2}\sqrt{x} - \frac{2}{x^2}$$

Thus the slope of the tangent line to the given curve at the point  $(1, 3)$  is given by

$$\left. \frac{dy}{dx} \right|_{x=1} = \frac{3}{2}\sqrt{1} - \frac{2}{1^2} = -\frac{1}{2}.$$

Therefore the equation of the tangent line is

$$\begin{aligned} y - (3) &= -\frac{1}{2}(x - 1) \\ y &= -\frac{1}{2}x + \frac{7}{2} \end{aligned}$$

**2** Find the rate of change  $\frac{dy}{dx}$  for the function  $y = (x^2 + 3x - 2)(x^3 - 4x + 3)$  when  $x = 1$ .

**Solution.** By the product rule, the derivative of  $y = (x^2 + 3x - 2)(x^3 - 4x + 3)$  with respect to  $x$  is given by

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} [(x^2 + 3x - 2)(x^3 - 4x + 3)] \\ &= (x^2 + 3x - 2)\frac{d}{dx} [x^3 - 4x + 3] + (x^3 - 4x + 3)\frac{d}{dx} [x^2 + 3x - 2] \\ &= (x^2 + 3x - 2)(3x^2 - 4) + (x^3 - 4x + 3)(2x + 3) \end{aligned}$$

Thus  $\frac{dy}{dx}$  when  $x = 1$  is

$$\begin{aligned} \left. \frac{dy}{dx} \right|_{x=1} &= (1^2 + 3 \cdot 1 - 2)(3 \cdot 1^2 - 4) + (1^3 - 4 \cdot 1 + 3)(2 \cdot 1 + 3) \\ &= 2(-1) + 0 \cdot 5 \\ &= -2 \end{aligned}$$