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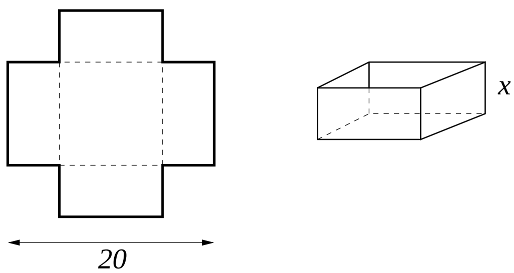
Solution of Quiz 3

1 An open box is to be made from a square piece of cardboard, 20 inches by 20 inches, by removing a small square from each corner and folding up the flaps to form the sides. Express the volume of the resulting box as a function of the length x of a side of the removed squares.



Solution. The resulting box has a square base of side length $20-2x$. Thus the volume of the resulting box is

$$V(x) = (20-2x)(20-2x)x = x(20-2x)^2.$$

2 Let f be the function defined by $f(x) = x^2 - 2$.

(a) Find the derivative of $f(x)$ with respect to x using the definition of the derivative.

Solution. According to the definition of the derivative,

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^2-2]-[x^2-2]}{h} \\ &= \lim_{h\to 0} \frac{(x^2+2xh+h^2-2)-(x^2-2)}{h} = \lim_{h\to 0}(2x+h) = 2x \end{aligned}$$

(b) Find the equation of the tangent line to the curve $y = f(x)$ at the point where $x = 1$.

Solution. when $x = 1$, the corresponding y coordinate on the given curve is $y = (1)^2 - 2 = -1$, so the point of tangency is $(1, -1)$.

Since $f'(x) = 2x$, the slope of the tangent line to the given curve at the point $(1, -1)$ is given by

$$f'(1) = 2(1) = 2.$$

Therefore the equation of the tangent line is

$$\begin{aligned} y-(-1) &= 2(x-1) \\ y &= 2x-3 \end{aligned}$$