

Solution of Quiz 1

1 What is the domain of the function $f(x) = \frac{x^2 + x - 2}{x - 2}$?

Solution. Since division by any nonzero number is possible, the domain of f is the set of all numbers satisfying $x - 2 \neq 0$. Thus the domain of f is the set of all real numbers x except $x = 2$.

2 Find all values of x such that $f(g(x)) = g(f(x))$, where $f(x) = x^2 + 2$ and $g(x) = x - 1$.

Solution. Replace x by $g(x) = x - 1$ in the formula for $f(x)$ to get

$$f(g(x)) = g(x)^2 + 2 = (x - 1)^2 + 2 = x^2 - 2x + 3.$$

Similarly, we replace x by $f(x) = x^2 + 2$ in the formula for $g(x)$ to get

$$g(f(x)) = f(x) - 1 = (x^2 + 2) - 1 = x^2 + 1.$$

By solving the equation $f(g(x)) = g(f(x))$, we get

$$\begin{aligned}x^2 - 2x + 3 &= x^2 + 1 \\-2x &= -2 \\x &= 1\end{aligned}$$

Thus $f(g(x)) = g(f(x))$ only when $x = 1$.

3 Find x and y intercepts of the function $f(x) = x^2 - 4x + 3$.

Solution. Since $f(0) = 3$, the y intercept is $(0, 3)$. To find the x intercepts, solve the equation $f(x) = 0$. Factoring, we find that

$$\begin{aligned}x^2 - 4x + 3 &= 0 \\(x - 1)(x - 3) &= 0\end{aligned}$$

Thus, the x intercepts are $(1, 0)$ and $(3, 0)$.