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Solution of Quiz 10

1 Find all real numbers that satisfy $2^{2-x} = 8^3$.

Solution. The given equation is satisfied if and only if

$$\begin{aligned} 2^{2-x} &= (2^3)^3 \\ 2^{2-x} &= 2^9 \\ 2 - x &= 9 \\ x &= -7 \end{aligned}$$

Thus, $2^{2-x} = 8^3$ if and only if $x = -7$.

2 Find the slope of the tangent line to the curve $f(x) = \ln(x^2+1)$ at the point where $x = 1$.

Solution. Since the derivative of $f(x)$ is

$$f'(x) = \frac{\frac{d}{dx}(x^2+1)}{x^2+1} = \frac{2x}{x^2+1},$$

the slope of the tangent line at the point where $x = 1$ is

$$f'(1) = \frac{2(1)}{(1)^2+1} = 1.$$

3 Use logarithmic differentiation to find the derivative of $f(x) = \dfrac{e^{2x}(2x+1)^3}{(3x-5)^4}$.

Solution. First take the derivative of both sides of the expression for f:

$$\begin{aligned} \ln f(x) &= \ln \frac{e^{2x}(2x+1)^3}{(3x-5)^4} \\ &= \ln e^{2x} + \ln(2x+1)^3 - \ln(3x-5)^4 \\ &= 2x + 3\ln(2x+1) - 4\ln(3x-5) \end{aligned}$$

Now use the chain rule for logarithms to differentiate both sides of this equation to get

$$\frac{f'(x)}{f(x)} = 2 + 3\left(\frac{2}{2x+1}\right) - 4\left(\frac{3}{3x-5}\right) = 2 + \frac{6}{2x+1} - \frac{12}{3x-5}$$

so that

$$\begin{aligned} f'(x) &= f(x)\left[2 + \frac{6}{2x+1} - \frac{12}{3x-5}\right] \\ &= \frac{e^{2x}(2x+1)^3}{(3x-5)^4}\left[2 + \frac{6}{2x+1} - \frac{12}{3x-5}\right] \end{aligned}$$