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Solution of Quiz 0

1 Find the interval or intervals consisting of all real numbers x that satisfy the given inequality.

(a) $|x-1| \leq 3$

SOLUTION. Rewrite the given inequality as

$$-3 \leq x-1 \leq 3$$

and add 1 to each term to get

$$-2 \leq x \leq 4.$$

(b) $|2-x| > 4$

SOLUTION. The given expression is true if and only if $2-x > 4$ or $2-x < -4$ which is equivalent to $x < -2$ or $x > 6$.

2 Evaluate the given expression without using a calculator.

(a) $8^{-2/3}$

SOLUTION. $8^{-2/3} = \sqrt[3]{8^{-2}} = \sqrt[3]{\frac{1}{8^2}} = \sqrt[3]{\frac{1}{(2^3)^2}} = \sqrt[3]{\frac{1}{2^6}} = \frac{1}{\sqrt[3]{2^6}} = \frac{1}{2^2} = \frac{1}{4}.$

(b) $\dfrac{3^3(3^2)^3}{9^4}$

SOLUTION. $\dfrac{3^3(3^2)^3}{9^4} = \dfrac{3^3 3^6}{(3^2)^4} = \dfrac{3^9}{3^8} = 3.$

3 Factor and simplify the given expressions as much as possible.

(a) $3x^4 - 12x^3$

SOLUTION. Because the common factor of $3x^4$ and $12x^3$ is $3x^3$, we have

$$3x^4 - 12x^3 = 3x^3(x-4).$$

(b) $\dfrac{3(x-2)^2(x+1)^2 - 2(x-2)(x+1)^3}{(x-2)^2}$

SOLUTION. First simplify the numerator to get

$$\begin{aligned}\frac{3(x-2)^2(x+1)^2 - 2(x-2)(x+1)^3}{(x-2)^2} &= \frac{(x-2)(x+1)^2[3(x-2) - 2(x+1)]}{(x-2)^2} \\ &= \frac{(x-2)(x+1)^2[3x-6-2x-2]}{(x-2)^2} \\ &= \frac{(x-2)(x+1)^2(x-8)}{(x-2)^2}\end{aligned}$$

and then cancel the common factor of $x-2$ from the numerator and the denominator to get

$$\frac{3(x-2)^2(x+1)^2 - 2(x-2)(x+1)^3}{(x-2)^2} = \frac{(x+1)^2(x-8)}{x-2}.$$

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Solution of Quiz 0

4 Solve the equation $6x^2 - 7x - 3 = 0$ by factoring.

SOLUTION. We want to find integers a, b, c, and d so that

$$\begin{aligned} 6x^2 - 7x - 3 = 0 &= (ax + b)(cx + d) \\ &= acx + (ad + bc)x + bd. \end{aligned}$$

so we must have

$$\begin{aligned} ac &= 6 \\ ad + bc &= -7 \\ bd &= -3 \end{aligned}$$

Since a, b, c, and d are all integers, there are a limited number of possibilities for the choices; namely
8 choices for the pair a, c: $1, 6; 2, 3; 3, 2; 6, 1; -1, -6; -2, -3; -3, -2$; and $-6, -1$
4 choices for the pair b, d: $1, -3; 3, -1; -1, 3$; and $-3, 1$

There are 32 possible ways of forming the expression $ad + bc$. One can see that the condition $ad + bc = -7$ is satisfied when $a = 2, b = -3, c = 3$, and $d = 1$, so that

$$6x^2 - 7x - 3 = (2x - 3)(3x + 1).$$

Thus the solution for the given equation is $x = \frac{3}{2}$ and $x = -\frac{1}{3}$.

5 Use the quadratic formula to solve the equation $x^2 - 2x - 2 = 0$.

SOLUTION. The given equation is a quadratic equation with $a = 1, b = -2$, and $c = -2$. Using the quadratic formula, we get

$$x = \frac{2 \pm \sqrt{2^2 - (-8)}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}.$$

6 Solve the following system of equations.

$$\begin{aligned} 3x^2 - 4y &= 0 \\ 2x - y &= 1 \end{aligned}$$

SOLUTION. Solve the second equation for y to get

$$y = 2x - 1$$

and substitute this into the first equation to eliminate y. This gives

$$\begin{aligned} 3x^2 - 4(2x - 1) &= 0 \\ 3x^2 - 8x + 4 &= 0 \\ (3x - 2)(x - 2) &= 0 \end{aligned}$$

from which it follows that $x = \frac{2}{3}$ or $x = 2$. If $x = \frac{2}{3}$, the second equation gives $y = \frac{1}{3}$ and if $x = 2$, the second equation gives $y = 3$. Hence the system has two solutions,

$$x = \frac{2}{3}, y = \frac{1}{3} \text{ and } x = 2, y = 3.$$