Math 493: Homework 4b – Anderson – Summer 2009

DUE: WEDNESDAY, JUNE 17, 2009

1. Write a finite-difference numerical scheme and use it to find the smallest 3 positive eigenvalues λ of the eigenvalue problem

$$\begin{aligned} u'' + \lambda(1+x^2)u &= 0, \\ u(0) &= 0, \\ u(1) &= 0. \end{aligned}$$

For each eigenvalue you find, plot the corresponding solution (eigensolution).

This scheme will involve setting up a generalized eigenvalue problem $Aw = \lambda Bw$ where A is a matrix associated with the finite difference discretization of the term u'' and B is a matrix that accounts for the coefficient $1 + x^2$. Note that if you were to replace $1 + x^2$ with 1 the B matrix would be the identity matrix and you would then have the eigenvalue problem $Aw = \lambda w$. You may wish to test this case out first since you can compare with an exact solution. You can use Matlab's command `[V,D]=eig(A,B)` to solve the generalized eigenvalue problem. Here D is a matrix with eigenvalues along the diagonal and V contains in its columns the corresponding eigenvectors.

Another useful command that will help order the eigenvalues that you obtain is `[s,i]=sort(x)` where s contains the entries in x sorted from smallest to largest and i contains the corresponding index identifying the location in the original vector x from where each component of s comes. For example, if $x = (2, 10, -3)$ then $s = (-3, 2, 10)$ and $i = (3, 1, 2)$.

In the process of obtaining the eigenvalues and eigenvectors please also check your results by increasing the number of points used in the spatial discretization. That is, verify (perhaps by making a table) that you have obtained convergence in your eigenvalues with respect to spatial discretization.