1. a. $u_{tt} = c^2 u_{xx}$

b. $u_{tt} = c^2 u_{xx} + u(1-u)$

c. $u_{tt} = c^2 t^3 u_{xx}$

d. $u_{tt} = c^2 u_{xx} u_t + 5t^2 + 1$

e. $u_{tt} = c^2 u_{xxt} + 1$

f. $u_t = -\frac{\partial^2}{\partial x^2}\left(\epsilon^2 u_{xx} + u - u^3\right)$

g. $u_t + x^2 u_{xx} = 3x + t^2$

h. $u_t + u\, u_{xx} + u_{xxt} = 0$

i. $\left(\frac{\partial^2 u}{\partial t^2}\right) - \left(\frac{\partial u}{\partial x}\right)^2 = 3u + xt$

j. $u_{ttt} + t^2 u = e^{x^2 t}\, u_{xx}$

Figure out the order, whether homogeneous, & whether linear for each equation.

2. For $u_{tt} = c^2 u_{xx}$ (the wave equation)

Show that $u(x,t) = \cos\left(\frac{n\pi x}{L}\right)\cos\left(\frac{cn\pi t}{L}\right)$ $n \in \mathbb{Z}$

is a solution.

Find some <u>homogeneous</u> boundary conditions which are satisfied by this sol'n.

(Think derivatives for initial conditions.)

3. Assume that

$y_1(x,t)$ solves ① $u_{tt} - c^2 u_{xx} = 0$ with homogeneous Dirichlet conditions.

Assume $z(x,t)$ solves ② $u_{tt} - c^2 u_{xx} = f(x,t)$ with homogeneous Dirichlet boundary cond's.

Then show that $w(x,t) = z(x,t) + a\, y_1(x,t)$, $a \in \mathbb{R}$ solves ② with homogeneous Dir. boundary conditions.

4. Assume that $y(x,t)$ & $z(x,t)$ both solve

$$\left.\begin{aligned} u_{tt} &= c^2 u_{xx} \\ u(0,t) &= f_1(t) \\ u(L,t) &= f_2(t) \\ u(x,0) &= f_3(x) \\ \frac{\partial u}{\partial t}(x,0) &= f_4(x) \end{aligned}\right\} \forall\ 0 < x < L\ ,\ t > 0$$

Show that $w(x,t) \equiv y(x,t) - z(x,t)$ solves

$$u_{tt} = c^2 u_{xx}$$

$\forall t > 0$: $u(0,t) = u(L,t)$ & $u(x,0) = u_t(x,0) = 0$ $\forall x \in [0,L]$

(Note that $u(x,t) \equiv 0$ $\forall x,t$ solves this last equation.)

Day 2 Problems 5/22/07

1. Run the linked matlab commands to see a plot of the solution of Laplace's equation discussed in class.

2. In problems 1.23-1.28, classify the equations. (Note I didn't say to do what they asked. You can if you want, but we didn't cover all of these topics.)

3. $0 = a\,u_{xx} + b\,u_{xy} + c\,u_{yy} + d\,u_{xz} + e\,u_{yz} + f\,u_{zz}$

   Write a matrix A which is symmetric such that we can classify this PDE as elliptic / hyperbolic / parabolic, using elliptic ⇔ definite / hyperbolic ⇔ indefinite with only one eigenvalue of opposite sign / parabolic ⇔ degenerate

4. assume that u satisfies Laplace's equation on $\Omega \subseteq \mathbb{R}^3$, and v is a cont'sly differentiable function so that $v \equiv 0$ on $\partial\Omega$.
Show that

$$\int_\Omega \nabla u \cdot \nabla v \, dx = 0$$

5. Let u solve the heat equation $u_t = k u_{xx}$, homogeneous Dirichlet boundary cond'ns on $[0, L]$
Show that

$$\boxed{E(t) = \int_0^L u^2(x,t)\, dx}$$

is decreasing as t grows.
ie. it is a decreasing function of t.

Day 3 Practice Problems

1. a. Show that $\left\{\cos\left(\frac{n\pi x}{L}\right)\right\}_{n=0}^{\infty}$ are orthogonal on $[0,L]$.

   b. Write $x = \sum c_n g_n(x) \leftarrow$ find c_n.

2. Show that $\left\{1,\ 1-x,\ 1-2x+\frac{x^2}{2}\right\}$

   are orthogonal wrt $\phi(x) = e^{-x}$ on $[0,\infty)$.

3. Find α & β s.t. $\{1, x, 1+\alpha x+\beta x^2\}$ is an ~~orthogonal~~ set on $[-1,1]$.

4. Let $T_n(x) = \cos(n \arccos(x))$.

   Show that $\{T_n\}$ is orthogonal wrt $\phi(x) = \frac{1}{\sqrt{1-x^2}}$

   on $[-1,1]$.

5. Try the following problems in the text:

   1.1, 1.2, 1.4, 1.7, 1.8

Daily Problems, Separation of Variables

Solve the following problems using separation of variables:

* a. $u_{xx} + u_{yy} - u + u_y = 0$ on $[0,1]\times[0,1]$ where $u(0,y) = u(1,y) = u(x,0) = 0$.

b. The vibrating string with fixed ends, with initial condition $u_t(x,0) = \sin(n\pi x/L)$ (on $x \in [0,L]$).

c. The heat equation on the space $[0,a]$ where $u_x(0,t) = u(a,t) = 0$, and $u(x,0) = x$.

d. Use separation of variables to write down a set of solutions to

$$u_{xx} + u_{yy} = 0 \quad \text{on } [0,a]\times[0,a]$$

with $u(x,0) = u(x,a) = u(a,y) = 0$,

& $u(0,y) = f(y)$