$$= \int_0^4 \int_0^2 \int_0^{x/2} \frac{4\cos(x^2)}{2\sqrt{z}}\, dy\, dx\, dz = \int_0^4 \int_0^2 \frac{4\cdot x/2}{2\sqrt{z}} \cos(x^2)\, dx\, dz$$

$$= \int_0^4 \frac{1}{2\sqrt{z}} \sin(x^2)\Big|_0^2\, dz = \sin(4)\cdot \int_0^4 \frac{1}{2\sqrt{z}}\, dz$$ ← improper integral!

$$= \left(\lim_{a\to 0^+} \int_a^4 \frac{1}{2} z^{-1/2}\, dz\right) \sin(4) = \left(\lim_{a\to 0^+} z^{1/2}\Big|_a^4\right) \sin(4) = 2\sin(4)$$

15.6 #17

$$\int_{-\pi/2}^{\pi/2}\int_{1}^{1+\cos\theta}\int_0^4 f(r,\theta,z)\, dz\, r\, dr\, d\theta$$

[illegible]

$0 \le z \le 4$

$-\pi/2 \le \theta \le \pi/2$ (quads I, IV)

$1 \le r \le 1+\cos\theta$

#21

$$\int_0^{\pi}\int_0^{\pi}\int_0^{2\sin\phi} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta = \int_0^{\pi}\int_0^{\pi} \frac{\rho^3}{3}\sin\phi \Big|_{\rho=0}^{\rho=2\sin\phi} d\phi\, d\theta$$

$$= \int_0^{\pi}\int_0^{\pi} \frac{8}{3}\sin^4\phi\, d\phi\, d\theta$$ use table to reduce or double angle formulas twice

$$= \int_0^{\pi} \left(\frac{8}{3}\cdot\frac{3}{8}\right)\pi\, d\theta = \boxed{\pi^2}$$

15.7 #7 $u = 3x+2y$, $v = x+4y$

$y = -\frac{3}{2}x + 1 \Leftrightarrow 3x+2y=2$

$y = -\frac{3}{2}x + 3 \Leftrightarrow 3x+2y=6$

$y = -\frac{1}{4}x \Leftrightarrow x+4y=0$

$y = -\frac{1}{4}x + 1 \Leftrightarrow x+4y=4$

Also $u - 3v = (2-12)y = -10y$

so $y = -\frac{u}{10} + \frac{3}{10}v$ $\quad \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 2/5 & -1/5 \\ -1/10 & 3/10 \end{vmatrix} = \frac{6-1}{50} = \boxed{\frac{1}{10}}$

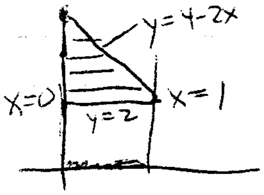
$2u - v = (6-1)x = 5x$ so $x = \frac{2}{5}u - \frac{1}{5}v$. Luckily $3x^2 + 14xy + 8y^2 = uv$

$$\int_2^6\int_0^4 \frac{uv}{10}\, dv\, du = \int_2^6 \frac{uv^2}{20}\Big|_0^4 du = \int_2^6 u\cdot\frac{16}{20}\, du = u^2\cdot\frac{16}{40}\Big|_2^6 = \frac{32\cdot 16}{40} = 64/5$$

CHAPTER 15 - THOMAS - HOMEWORK SOLUTION

15.1

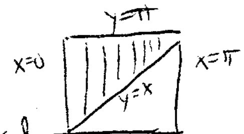
#21 $0 \le x \le 1$, $2 \le y \le 4-2x$



so $2 \le y \le 4$
$0 \le x \le \frac{4-y}{2}$
Left, Right

$$\int_2^4\int_0^{\frac{4-y}{2}} dx\,dy$$

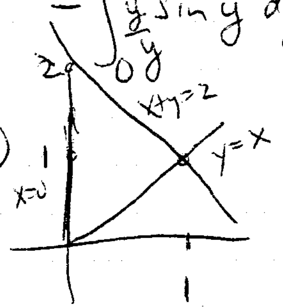
#31 $$\int_0^\pi\int_x^\pi \frac{\sin y}{y}\,dy\,dx = \int_0^\pi\int_0^y \frac{\sin y}{y}\,dx\,dy$$



$x \le y \le \pi$
$0 \le x \le \pi$
Image $0 \le y \le \pi$
$0 \le x \le y$

$$= \int_0^\pi \frac{y}{y}\sin y\,dy = -\cos y\Big|_0^\pi = \boxed{2}$$

#41



Slice in x first:
(First in y uses 2 pieces)
$$\int_0^1\int_x^{2-x}(x^2+y^2)\,dy\,dx$$

$$= \int_0^1\left[x^2y + y^3/3\Big|_{y=x}^{y=2-x}\right]dx$$

$$= \int_0^1\left[x^2(2-x) + \frac{(2-x)^3}{3} - x^3 - x^3/3\right]dx = \int_0^1\left[2x^2 - x^3 + \frac{(2-x)^3}{3} - \frac{4}{3}x^3\right]dx$$

$$= \frac{2}{3}x^3 - \frac{7}{12}x^4 - \frac{(2-x)^4}{12}\Big|_0^1 = 2/3 - \frac{7}{12} - \frac{1}{12} + \frac{16}{12}$$

$$= \boxed{4/3}$$

15.3 #15 $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2+y^2+1)\,dx\,dy$ Polar whole disk $x^2+y^2 \le 1$

$\int_0^{2\pi}\int_0^1 \ln(r^2+1)\,r\,dr\,d\theta$ Now $\int_0^1 r\ln(r^2+1)\,dr = \int_1^2 \frac{1}{2}\ln u\,du$ $u=r^2+1$, $du=2r\,dr$

$= \frac{1}{2}u\ln u - \frac{1}{2}u \Big|_1^2 = \frac{1}{2}2\ln 2 - 0 - \frac{1}{2}$

Answer $\boxed{2\pi\cdot\left(\ln 2 - \frac{1}{2}\right)}$

#29 Avg height of hemisphere $\bar{z} = \iint z\,dA \Big/ \iint 1\,dA$

$= \iint_{x^2+y^2\le a^2} \sqrt{a^2-x^2-y^2}\,dx\,dy \Big/ \pi a^2 \leftarrow$ area of circle!

But in polar: $\iint_{x^2+y^2\le a^2} \sqrt{a^2-x^2-y^2}\,dx\,dy = \int_0^{2\pi}\int_0^a \sqrt{a^2-r^2}\,r\,dr\,d\theta$

$= \int_0^{2\pi} -\frac{1}{3}(a^2-r^2)^{3/2}\Big|_{r=0}^{r=a} d\theta = \int_0^{2\pi}\left[-\frac{1}{3}\cdot 0 + \frac{1}{3}\cdot a^3\right] d\theta = \frac{2\pi}{3}a^3$

So $\bar{z} = \frac{2\pi}{3}a^3 \Big/ \pi a^2 = \boxed{\frac{2}{3}a}$

15.4 #23 Volume: $z=y^2$ top, $z=0$ bottom. COULD DO OTHER ORDERS

So $\int_0^1\int_{-1}^1 y^2\,dy\,dx = \int_0^1\left[\frac{y^3}{3}\Big|_{-1}^1\right]dx = \int_0^1 \frac{2}{3}\,dx = \boxed{\frac{2}{3}}$

#41 $\int_0^4\int_0^1\int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\,dx\,dy\,dz$

$2y \le x \le 2$
$0 \le y \le 1$
reversed to $0 \le x \le 2$
$0 \le y \le x/2$