Name _____ *ANSWER KEY* _____

Math 554-B01, Summer 2014, Test 3, O'Beirne

Answer all questions. Show all work leading to the answer for credit. You must work alone. The Honor Code is in effect.

1. A ten-year $1,000 par value bond bearing a 10% coupon rate payable semi-annually and redeemable at $1,100 is bought to yield 8% compounded semi-annually. Find the purchase price. _____ 1181.54 _____

Work:

$$PMT \quad 50$$
$$FV \quad 1100$$
$$i \quad 4\%$$
$$m \quad 20$$
$$CPT \quad PV = 1,181\ 54$$

2. An n-year $1,000 par value bond has a redemption value of $1,000 and has a coupon rate of 12% compounded semi-annually. It is bought at a price to yield 10% compounded semi-annually. If the term of the bond is doubled, the price will increase by $48. Find the price of this n-year bond. _____ 1080 or 1120 _____

Work:

$$P = 1200 + (1000 - 1200)v^n$$
$$= 1200 - 200v^n$$
$$P + 48 = 1200 - 200v^{2n}$$
$$\therefore \ 1248 - 200v^n = 1200 - 200v^{2n}$$
$$200v^{2n} - 200v^n + 48 = 0$$
$$v^n = .6 \quad \text{or} \quad .4$$
$$\therefore \ P = 1200 - 200v^n = 1080 \quad \text{or} \quad 1120$$

3. For a $1 bond the coupon rate is 120% of the yield rate and the premium is p. For another $1 bond with the same number of coupons and the same yield rate, the coupon rate is 60% of the yield rate. Express the price of the second bond as a function of p. _____ $1 - 2p$ _____

Work:

$$P_1 = 1 + p = 1 + (1.2i - i)\,a_{\overline{n}|} = 1 + .2i\,a_{\overline{n}|}$$
$$\longrightarrow p = .2i\,a_{\overline{n}|}$$
$$P_2 = 1 + (.6i - i)\,a_{\overline{n}|} = 1 - .4i\,a_{\overline{n}|} = 1 - 2p$$

4. A $100 bond with annual coupons is redeemable at par at the end of 15 years. At a purchase price of $90 the yield rate is exactly 1% more than the coupon rate. Find the yield rate of the bond. $\underline{5.5565\ \%}$

Work:

$$90 = 100 \left[1 - .01\, a_{\overline{15}|i} \right]$$

$$\longrightarrow a_{\overline{15}|i} = 10$$

$$\longrightarrow i = 5.5565\% \quad \text{by calculator}$$
$$(PMT = 1 \quad N = 15 \quad PV = -10 \quad CPT\ i)$$

5. A $1000 par value 10% bond with quarterly coupons is callable five years after issue. The bond matures for $1000 at the end of ten years and is sold to yield a nominal rate of 8% compounded quarterly under the assumption that the bond will not be called. Find the redemption ~~rate~~ at the end of five years that would provide the purchaser the same yield rate.

amount

~~redemption~~ $\underline{1081.77}$

Work:

$$P = 1000\,(1.02)^{-40} + 25\, a_{\overline{40}|2\%} = 452.89 + 683.89 = 1136.78$$

$$1136.78 = C\,(1.02)^{-20} + 25\, a_{\overline{20}|2\%}$$

$$\longrightarrow C = 1136.78\,(1.02)^{20} - 25\, S_{\overline{20}|2\%} = 1689 - 607.43 = 1081.77$$

6. A common stock pays annual dividends at the end of each year. The earnings per share in the year just ended were $8. Earnings are expected to grow at the rate of 10% each year in the future. The percentage of earnings paid out as a dividend will be 0% for the next 4 years and 50% thereafter. Find the theoretical price of the stock that would yield the investor 12% effective. $\underline{204.69}$

Work:

EPS	8	8(1.1)	8(1.1)²	8(1.1)³	8(1.1)⁴	8(1.1)⁵	8(1.1)⁶	···
%		0	0	0	0	50	50	···
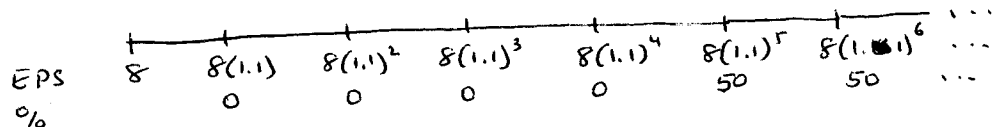
$$PV = V^5\left(8(1.1)^5(.50) \right) + V^6\left(8(1.1)^6(.50) \right) + \cdots$$

$$= \frac{V^5\left(8(1.1)^5(.50) \right)}{1 - V(1.1)} = \frac{.5674\,(6.44204)}{.017857} = 204.69$$

7. An investor makes a single deposit of $20,000 into Fund A for 10 years which earns 6% effective rate of interest payable directly to the investor each year. During the first 5 years the interest payments can only be reinvested into Fund B which earns 5% effective over the course of the ten years. During the second five years the interest payments can only be reinvested into Fund C which earns 4% effective. Find the total accumulated value in Funds A, B, and C combined at the end of ten years to the nearest dollar. Find the overall yield rate achieved by the investor. $\underline{5.744\ ^{\circ}/_{o}}$

Work:

$$20{,}000 + 12\ S_{\overline{5}|\,5\%}\,(1.05)^5 + 1200\ S_{\overline{5}|\,4\%}$$

$$= 20{,}000 + 6630.76\,(1.05)^5 + 6499.59$$

$$= 34{,}962.31$$

$$20{,}000\,(1+i)^{10} = 34{,}962.31 \quad \rightarrow \quad i = 5.744\%$$

8. A loan of $10,000 is being repaid with payments of $1,000 at the end of each year for 25 years. If each payment is reinvested at 6% effective, find the effective annual rate of interest earned over the 25-year period. $\underline{7.05\%}$

Work:

$$1000\ S_{\overline{25}|\,6\%} = 54{,}864.51$$

$$10{,}000\,(1+i)^{25} = 54{,}864.51$$

$$(1+i)^{25} = 5.486451$$

$$1+i = 1.07462$$

9. A $100 par (face) value 12-year bond with 8% semiannual coupons is selling for $120. If the coupons can only be reinvested at 6% compounded semiannually, compute the overall yield rate achieved by a bond purchaser over the 12-year period. $\underline{5.778\ ^{\circ}/_{o}}$

Work:

$$4\ S_{\overline{24}|\,3\%} + 100$$

$$= 137.71 + 100 = 237.71$$

$$120\,(1+j)^{24} = 237.71$$

$$(1+j)^{24} = 1.9809$$

$$1+j = 1.02889$$

$$j = .02889$$

$$\therefore\ i = 2(.02889) = .05778$$

10. On January 1 an investment account is worth $100,000. On March 1 the value has increased to $110,000 and $25,000 of new principal is deposited. On September 1 the value has declined to $125,000 and $40,000 is withdrawn. On January 1 of the following year the account is again worth $100,000. Calculate the yield rate by the dollar-weighted method and by the time-weighted method. $i_{DW} =$ __13.95%__ $i_{TW} =$ __19.83%__ ✓

Work:

$$I = B - A - C = 100 - 100 - (-15) = 15$$

$$i_{DW} = \frac{15}{100(1) + 25(\frac{5}{6}) - 40(\frac{1}{3})} = \frac{15}{100 + 20.83 - 13.3} = \frac{15}{107.5} = 13.95\%$$

$$i_{TW} = \frac{110}{100} \cdot \frac{125}{135} \cdot \frac{100}{85} - 1 = .198257 = 19.8257\%$$

11. Use the following table excerpt to find the dollar amount of interest which $10,000 invested at time z will earn over the 3rd through 6th years of investment (inclusive).

__~~3327.8~~ 4378__ ✓

Calendar Year   Investment Year Rates

| Of Investment	i_1	i_2	i_3	i_4	Portfolio Rate
Z →	8% →	8.10% →	8.20% →	8.25% →	8.20% ↓
Z+1	9%	9.10%	9.20%	9.25%	8.50%
Z+2	10%	10.10%	10.20%	10.25%	8.75%

(over and down)

Work:

After 2 years: $10,000(1.08)(1.081) = 11,674.80$

After 6 years: $11,674.80(1.082)(1.0825)(1.082)(1.085) = 16,053.20$

Difference: $16053 - 11,675 = 4,378$

12. Payments of $100 now and $110 two years from now are equivalent to $209.80 one year from now at either rate i or j. Find the absolute difference of the two rates. __.04__ ✓

Work:

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100                      110
├──────────┬──────────┤
↑           209.8
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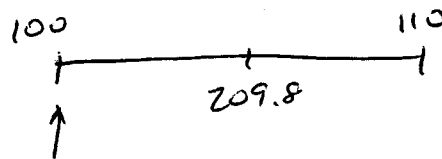
$$100 + 110v^2 = 209.8v$$

Solve for v → $i_1 =$
       $i_2 =$

$|i_1 - i_2| = .04$