NAME ANSWER KEY

Math 554-B01, Summer 2014, Test #2, O'Beirne

Answer all questions. Show all work for full credit. You must work alone. The Honor Code is in effect.

4 #4 1. An annuity-immediate that pays 500 quarterly for the next 15 years costs 16,000. Calculate the nominal interest rate compounded monthly earned by this investment. 9.32 %

Work:

$16000 = 500\, a_{\overline{60}|j} \rightarrow j = 2.349\%$

$(1+k)^3 = 1.02349$

$k = .00777$

$12k = 9.32\%$

4 #14 2. $15\, a_{\overline{n}|}^{(2)} = 12\, a_{\overline{2n}|}^{(2)} = 90\, s_{\overline{1}|}^{(2)}$ What is i?

$i = .125$

Work:

$15 \frac{1-v^n}{i^{(2)}} = 12 \frac{1-v^{2n}}{i^{(2)}}$

$15 - 15v^n = 12 - 12v^{2n}$

$12v^{2n} - 15v^n + 3 = 0$

$4v^{2n} - 5v^n + 1 = 0$

$(4v^n - 1)(v^n - 1) = 0$

$\rightarrow v^n = \frac{1}{4}$

$15 \frac{1-v^n}{i^{(2)}} = 90 \frac{i}{i^{(2)}}$

$15 - 15v^n = 90i$

$i = \frac{15(1-v^n)}{90} = \frac{15(1-\frac{1}{4})}{90} = .125$

4 #17 3. There is \$50,000 in a fund which is accumulating at 4% per year compounded continuously. If the money is withdrawn continuously at the rate of \$3,000 per year, how long will the fund last? 27.465 years

Work:

$50,000 = 3,000\, \bar{a}_{\overline{n}|\delta=.04}$

$50,000 = 3,000 \left(\frac{1-e^{-.04n}}{.04}\right)$

$\frac{50,000(.04)}{3,000} = 1 - e^{-.04n}$

$\frac{2}{3} = 1 - e^{-.04n}$

$e^{-.04n} = \frac{1}{3}$

$-.04n = \ln(\frac{1}{3})$

$n = 27.465$

4 #18 4. If $\bar{a}_{\overline{n}|} = 3$ and $\bar{s}_{\overline{n}|} = 8$ what is δ?

$\delta = 5/24 = .2083\overline{3}$

Work:

$$\bar{a}_{\overline{n}|} = \frac{1-v^n}{\delta} = 3 \qquad v^n = 1 - 3\delta$$

$$\bar{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{\delta} = 8 \qquad (1+i)^n = 1 + 8\delta$$

So $1 - 3\delta = \frac{1}{1+8\delta}$

$1 + 5\delta - 24\delta^2 = 1$

$5\delta = 24\delta^2$

$5 = 24\delta$

$\delta = 5/24$

4 #25 5. A perpetuity-immediate has annual payments of 1, 4, 7, 10, … If the present value of the fifth and sixth payments are the same, what is the present value of the perpetuity? $18\frac{2}{3} = 60.66\overline{6}$

Work:

$$13v^5 = 16v^6$$

$$13 = 16v$$

$$v = 13/16$$

$$A\,PV = v + 4v^2 + 7v^3 + 10v^4 + \cdots$$

$$Av = \qquad v^2 + 4v^3 + 7v^4 + \cdots$$

$$A - Av = v + 3v^2 + 3v^3 + 3v^4 + \cdots$$

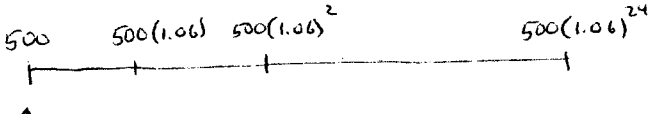
$$A(1-v) = v + 3\left(\frac{v^2}{1-v}\right)$$

$$A = \frac{v}{1-v} + \frac{3v^2}{(1-v)^2} = \frac{13/16}{3/16} + \frac{3(13/16)^2}{(3/16)^2} = \frac{13}{3} + \frac{169}{3} = \frac{182}{3} = 60.6\overline{6}$$

4 #28 6. What is the present value of an 25-year annuity-due with annual payments if the first payment of \$500 is payable now and each subsequent payment is 6% greater than the previous payment. Assume i = 9%.

9124.77

Work:

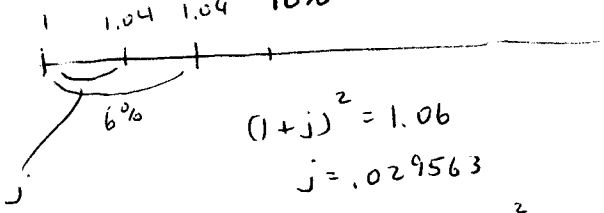


$$A = 500 + 500\left(\frac{1.06}{1.09}\right) + 500\left(\frac{1.06}{1.09}\right)^2 + \cdots + 500\left(\frac{1.06}{1.09}\right)^{24}$$

$$= \frac{500 - 500\left(\frac{1.06}{1.09}\right)^{25}}{1 - \frac{1.06}{1.09}} = \frac{251.14052}{.027523} = 9124.77$$

4 #38 7. A perpetuity provides payments every six months starting today. The first payment is 1 and each subsequent payment is 4% greater than the previous payment. Find the present value of the perpetuity if the effective rate of interest is 10% per year. not defined (∞)

Work:



$(1+j)^2 = 1.06$

$j = .029563$

$A = 1 + \frac{1.04}{1.029563} + \left(\frac{1.04}{1.029563}\right)^2 + \ldots$

$\frac{1}{1-r}$ where $r = \left(\frac{1.04}{1.029563}\right) > 1$ So does not converge!

PV not defined

5 #1 8. A loan of \$10,000 is being repaid with payments at the end of each quarter for 10 years at 8% compounded quarterly. Find the outstanding balance at the end of the third year. 7,779.53

Work:

$i^{(4)} = .08 \quad j = .02$

$OB = P a_{\overline{28}|2\%}$ where P is 365.5575

$= 7,779.53$ (calculator)

5 #3 9. A loan is being repaid with quarterly payments of \$1,000 at the end of each quarter for six years at 8% compounded quarterly. Find the amount of principal in the sixth payment. 686.43

Work:

After 5 payments OB is $1000\, a_{\overline{19}|2\%} = 15,678.46$

int in pmt 6 is $.02(15678.46) = 313.57$

prin in pmt 6 is $1000 - 313.57 = 686.43$

5. #14 10. A 35-year loan is to be repaid with equal payments at the end of each year. The amount of interest in the 8th payment is \$126. The amount of interest in the 22nd payment is \$105. Calculate the amount of interest paid in the 29th payment. 72.55

Work:

$$\frac{P(1-v^{28})}{P(1-v^{14})} = \frac{126}{105} = 1.2 = 1+v^{14}$$

$v^{14} = .2$

$v^7 = .4472$

$P(1-v^{14}) = P(.8) = 105$

$P = 131.25$

$P(1-v^7) = 131.25(1-.4472) = 72.55$

5. #26 11. A borrows \$10,000 for five years at 12% compounded semi-annually. He replaces the principal by means of deposits at the end of every year for five years into a sinking fund which earns 9% effective. Find the total dollar amount which A must pay over the five year period to completely repay the loan. 14,354.62

Work:

Int is 600 every 6 months

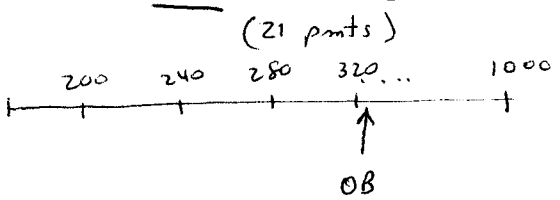
1) $s_{\overline{5}|9\%} = 10{,}000$

D = 1670.92457

6,000 + 8354.62 = 14,354.62

5 #29 12. A loan is repaid with payments which start at \$200 the first year and increase by \$40 per year until a payment of \$1,000. is made at which time payments cease. If interest is 5% effective, find the amount of interest in the fifth payment . 357.21

Work:



Int is 5% of 7144.28 = 357.21

$360v + 400v^2 + \ldots + 1000v^{17}$

$320v + 320v^2 + \ldots + 320v^{17}$
$40v + 80v^2 + \qquad + 680v^{17}$

$= 320(a_{\overline{17}|5\%}) + 40(Ia)_{\overline{17}|5\%}$

$= 3{,}607.70 + 40\left(\frac{\ddot{a}_{\overline{17}|5} - 17v^{17}}{.05}\right)$

$= 3{,}607.70 + 40\left(\frac{11.83777 - 7.417}{.05}\right) = 3607.70 + 3536.58 = 7{,}144.28$