NAME ANSWER KEY

Math 554-B01, Summer 2014, Test 1, O'Beirne

Answer as many questions as you can in the time allotted. You must show the work that leads to your answer for credit. If using a calculator to calculate annuities show inputs. You must work alone. The test is closed book. The Honor Code is in effect. Time allowed: 75 minutes.

1. Consider the amount function $A(t) = t^2 + 3t + 5$. Find i_5 $\underline{36.\overline{36}\,\%}$

Work:

$$A(4) = 16 + 12 + 5 = 33$$
$$A(5) = 25 + 15 + 5 = 45$$
$$i_5 = \frac{45-33}{33} = \frac{12}{33} = .\overline{36}$$

2. It is known that \$800 deposited for 3 years will earn \$284 in compound interest. Find the accumulated value of \$2,000 deposited at the same rate of compound interest for four years.

$\underline{2{,}998.81}$

Work:

$$\frac{800 + 284}{800} = 1.355 = (1+i)^3$$
$$1.10657 = 1+i$$
$$1.4994 = (1+i)^4$$
$$2000(1+i)^4 = 2{,}998.81$$

3. Express $i^{(5)}$ as a function of $d^{(3)}$ $\underline{i^{(5)} = 5\left(\left(1 - \frac{d^{(3)}}{3}\right)^{-3/5} - 1\right)}$

Work:

$$\left(1 + \frac{i^{(5)}}{5}\right)^5 = \left(1 - \frac{d^{(3)}}{3}\right)^{-3}$$
$$\left(1 + \frac{i^{(5)}}{5}\right) = \left(1 - \frac{d^{(3)}}{3}\right)^{-3/5}$$
$$\frac{i^{(5)}}{5} = \left(1 - \frac{d^{(3)}}{3}\right)^{-3/5} - 1$$
$$i^{(5)} = 5\left(\left(1 - \frac{d^{(3)}}{3}\right)^{-3/5} - 1\right)$$

1.37 4. Find the level effective rate of interest over a three year period which is equivalent to an effective rate of discount of 6% the first year, 5% the second year and 4% the third year.

~~[illegible]~~ 5.267 %

Work:

$$\frac{1}{1-.06} \cdot \frac{1}{1-.05} \cdot \frac{1}{1-.04} =$$

$$(1.0638298)(1.0526315)(1.0416667) = 1.16648 = (1+i)^3$$

$$1+i = 1.05267$$

1.49 5. Find $\frac{dv}{d\delta}$ in terms of v

$-v$

Work:

$$e^{\delta} = 1+i$$

$$e^{-\delta} = \frac{1}{1+i} = v$$

$$\frac{dv}{d\delta} = \frac{d}{d\delta}(e^{-\delta}) = e^{-\delta}(-1) = -e^{-\delta} = -v$$

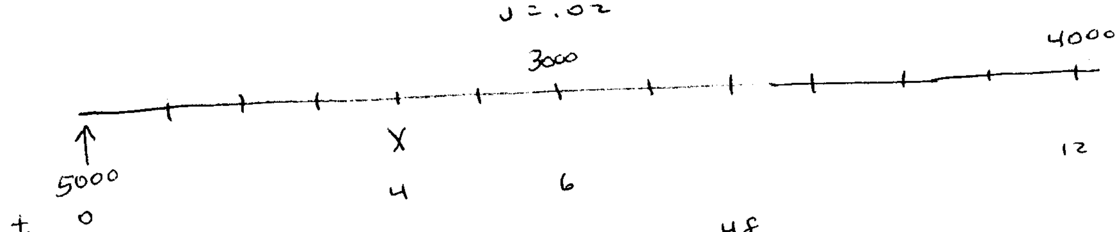
6. In return for a payment of \$3,000 at the end of six years and \$4,000 at the end of twelve years an investor agrees to pay \$5,000 immediately and to make an additional payment at the end of

2.1 four years. Find the amount of the additional payment if $i^{(4)} = 0.08$

-2180.93

Work:

$j = .02$



$$5000 + Xv^{16} = 3000v^{24} + 4000v^{48}$$

$$5000 + X(.728446) = 3000(.621721) + 4000(.386538)$$

$$5000 + 1546.15$$

$$5000 + .728446X = 1865.16 + 1546.15$$

$$X = \frac{-1588.69}{.728446} = -2180.93$$

7. Find how long $1,000 should be left to accumulate at 5% effective in order that it will accumulate to twice the accumulated value of another $1,000 deposited at the same time at 3% effective. 36.04 ~~51.048~~ years

Work:

5% 1000

3% 1000

$$1000(1.05)^n = 2 \cdot 1000(1.03)^n$$
$$(1.05)^n = 2(1.03)^n$$
$$\left(\frac{1.05}{1.03}\right)^n = 2$$
$$n \ln\left(\frac{1.05}{1.03}\right) = \ln 2$$
$$n = \cancel{51.248}\ 36.04$$

8. Fund A accumulates at a rate of 12% compounded monthly. Fund B accumulates at a force of interest δ_t = t/10. At time t equal deposits are made into each fund. Find the next time that the two funds have equal balances. $t = 2.388$

Work:

$$(1.01)^{12n} = e^{\int_0^n t/10\, dt}$$
$$(1.01)^{12n} = e^{n^2/20}$$
$$12n \ln(1.01) = n^2/20$$
$$\text{if } n \neq 0 \quad 12 \ln(1.01) = n/20$$
$$n = 240 \ln(1.01)$$
$$n = 2.388$$

9. It is known that a deposit of $1,000 will accumulate to $1,750 at the end of ten years. If it is assumed that the deposit earns simple interest at the rate ~~if~~ of i during the first year, 2i during the second year, ……., 10i during the tenth year what is i? ~~13.63%~~ 1.36%

Work:

$$1000(\cancel{1+i}\ i) + 1000(\cancel{1+2i}\ 2i) + \ldots + 1000(10i) = 750$$
$$1000i(1+2+\ldots+10) = 750$$
$$1000i = \frac{750}{55}$$
$$i = \frac{750}{55000} = \cancel{13.63\%}\ 1.36\%$$

10. Fund A accumulates at 4% effective and Fund B accumulates at 6% effective. At the end of twenty years the total of the two funds is \$1600. At the end of ten years Fund A is half of Fund B. What is the total of the two funds at the end of five years? 732.19

Work:

$A(1.04)^{20} + B(1.06)^{20} = 1600$

$2A(1.04)^{10} = B(1.06)^{10}$

$A(1.04)^{5} + B(1.06)^{5} = ?$

$259.80 + 472.39$

732.19

$2.1911A + 3.2071B = 1600$

$2.9605A - 1.79084B = 0$

$5.301768A - 3.2071B = 1600$

$7.492869A = 1600$

$A = 213.536$

$B = 353.00$

11. You wish to accumulate \$50,000 at the end of 16 years. If you deposit \$1,200 at the end of each of the first 8 years and \$X at the end of each year for the second 8 years, what is X if the fund earns 8% effective? 2479.98

Work:

let $X = 1200 + Y$

$1200\, s_{\overline{16|}\,8\%} + Y s_{\overline{8|}\,8\%} = 50{,}000$

$36{,}389.14 + 10.6366Y = 50{,}000$

$Y = 1279.98$

$X = 1200 + 1279.98$

$= 2479.98$

12. Payments of \$100 per quarter are made from June 7, 2010 through December 7, 2022 inclusive. If the nominal rate of interest compounded quarterly is 8% find the

Present value on September 7, 2008 2822.67

Current value on March 7, 2017 5534.33

Accumulated value on June 7, 2024 9828.13

Work:

t=0

100 100

9/7/08 6/7/10 12/7/22

51 pmts

$v^6\, 100\, a_{\overline{51|}\,2\%}$

$= (.88797)(3178.78)$

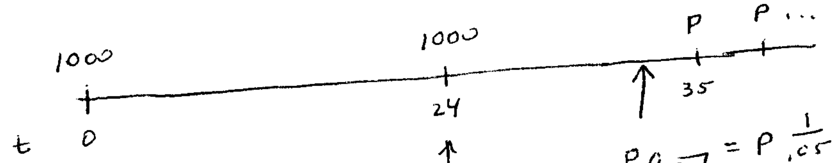
$= 2822.67$

$2822.67(1.02)^{34} = 5534.33$

$2822.67(1.02)^{63} = 9828.13$

13. Deposits of \$1,000 are placed into a fund at the beginning of each year for the next 25 years. After 35 years payments commence and continue forever with the first payment at the end of the 35^{th} year. Find the amount of each payment if interest is 5% effective. <u>3887.12</u>

Work:



$P a_{\overline{\infty}|} = P \frac{1}{.05} = 20P$

$1000\, s_{\overline{25}|5\%}$

$47,727.10$

$47727(1.05)^{10} = 20P \qquad 3,887.12 = P$

14. The present values of the following three annuities are equal:
(1) a perpetuity due paying 1 at the <u>beginning</u> of each year at an annual effective interest rate of 5%. (2) a 40-year annuity immediate paying 1 each year at an annual effective rate of j%. (3) an n-year annuity immediate paying 1 each year at an annual effective rate of j-1%. What is n?

<u>30.819</u>

Work:

$PV = \frac{1}{d} = 21 \qquad d = \frac{i}{1+i} = \frac{5}{105} = \frac{1}{21}$

$a_{\overline{40}|j} = 21 \rightarrow j = 3.6086\%$

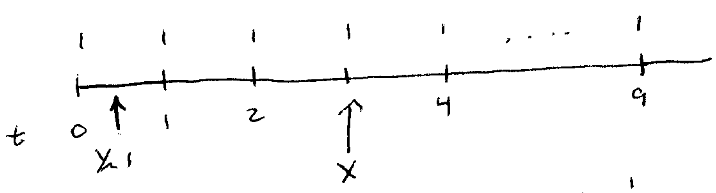
$j - 1 = 2.6086\%$

$a_{\overline{n}|2.6086} = 21 \rightarrow n = 30.819$

15. Given that X is the current value at time 3 of a 10-year annuity due of 1 per year, and the annual effective rate of interest for year t is $\frac{1}{6+t}$. Find X. Express your answer in simplified summation (Σ) form with an index of t.

$\sum_{t=7}^{16} \frac{10}{t}$

Work:



$\cancel{(1+\frac{1}{6+t})^3 + (1+\frac{1}{6+t})^2 + (1+\frac{1}{6+t})^1 + 1 + \frac{1}{1+1}}$

$(1+\frac{1}{7})(1+\frac{1}{8})(1+\frac{1}{9}) = \frac{8}{7}\frac{9}{8}\frac{10}{9} = \frac{10}{7}$

$(1+\frac{1}{8})(1+\frac{1}{9}) = \frac{9}{8}\cdot\frac{10}{9} = \frac{10}{8}$

$(1+\frac{1}{9}) = \frac{10}{9}$

$1 = \frac{10}{10}$

$(1+\frac{1}{10})^{-1} = \frac{10}{11}$

etc

$X = \frac{10}{7} + \frac{10}{8} + \frac{10}{9} + \dots + \frac{10}{16} = \sum_{t=7}^{16} \frac{10}{t}$