MATH 114, Section 002 Name `Solutions`
Spring, 2009
Exam 3

1. How many ways are there of choosing first, second, and third places and a set of 4 honorable mentions from 100 entrants at a prettiest pig contest? All four honorable mentions are equivalent, and no entrant can win more than one of the awards; for example, no pig can win both first place and an honorable mention.

   There are $P(100, 3) = 100 \cdot 99 \cdot 98$ ways of choosing the first, second, and third place winners. There are $C(97, 4) = \frac{97!}{4!93!}$ ways of choosing the 4 honorable mentions from the remaining 97 pigs. Therefore, there are $100 \cdot 99 \cdot 98 \left(\frac{97!}{4!93!}\right)$ ways of making the selections.

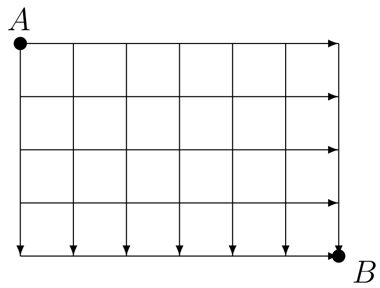
   Answer:
   $$100 \cdot 99 \cdot 98 \left(\frac{97!}{4!93!}\right)$$

2. How many ways are there of arranging the letters of the word STREAMING in such a way that the vowels (A, E, I) are in alphabetical order. (For example, ASTREGNIM is such an arrangement, but ESTRAGNIM is not.) Note that the word STREAMING has 9 letters.

   First choose 3 of the 9 positions in which to place the vowels. There are $C(9, 3) = \frac{9!}{3!6!}$ ways of doing that. Having chosen the three positions, there is only one way to place the vowels in alphabetical order. There are $P(6, 6) = 6!$ ways of arranging the consonants in the remaining 6 positions. Therefore, there are $\left(\frac{9!}{3!6!}\right) 6! = \frac{9!}{3!} = 60480$ arrangements of the given type.

   Answer:
   $$\frac{9!}{3!}$$

3. In the map below, how many routes are there from point A to point B? Assume that all roads are one-way, either west-to-east or north-to-south.



Every route from A to B can be viewed as a sequence of 10 symbols, namely, 6 E's and 4 S's. Therefore, the number of routes is the number of 10-bit strings consisting of 6 E's and 4 S's. There are $C(10,6) = \frac{10!}{6!4!} = 210$ such sequences.

Answer:
210

4. Find the coefficient of x^3 in the expansion of $(x+2)^7$.

The term containing x^3 is $\binom{7}{3} x^3 2^4 = \left(\frac{7!}{3!4!}\right) x^3 \cdot 16 = 35 \cdot 16x^3 = 560x^3$. Therefore, the coefficient is 560.

Answer:
560

5. How many arrangements are there of the letters of the word ADDRESSES? Note that the word ADDRESSES has 9 letters.

`The word ADDRESSES has 1 A, 2 D's, 1 R, 2 E's, and 3 S's. Therefore, the number of arrangements is` $\binom{9}{1,2,1,2,3} = \frac{9!}{1!2!1!2!3!} = 15120$.

Answer:
$\frac{9!}{1!2!1!2!3!}$

6. Find the coefficient of $x^6y^2z^3$ in the expansion of $(x+y+z)^{11}$.

`By the Multinomial Theorem, the coefficient is` $\binom{11}{6,2,3} = \frac{11!}{6!2!3!} = 4620$.

Answer:
$\frac{11!}{6!2!3!}$

7. A coin is tossed three times and the sample space is $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

(a) Find the subset E of S which gives the event "An odd number of heads comes up."

Answer:
$\{HHH, HTT, THT, TTH\}$

(b) Describe in words the event $\{HHH, HHT, HTH, HTT\}$.

Answer:
`The first toss comes up heads.`

8. Suppose that 10 coins are tossed. Let E be the event "At least 5 of the coins come up heads" and let F be the event "The first coin comes up heads." Using correct grammar, describe each of the following events. Your answer should be an English sentence.

   (a) $E \cap F$

   Answer:
   `At least 5 coins, including the first, come up heads.`

   (b) $E \setminus F$

   Answer:
   `At least 5 coins come up heads, but the first does not come up heads.`

9. Suppose that $S = \{a, b, c, d, e, f, g\}$ is an equiprobable space. Find $Pr(\{a, c, f\})$.

   Since S is an equiprobable space with 7 elements, the probability of each simple event is $\frac{1}{7}$. Since the set $\{a, c, f\}$ has three elements, $Pr(\{a, c, f\}) = \frac{3}{7}$.

   Answer:
   $\frac{3}{7}$

10. A fair coin is tossed twice and we are interested in the number of times heads come up. We use the sample space $S = \{0 \text{ heads}, 1 \text{ head}, 2 \text{ heads}\}$. Find the probability distribution for S.

S is not equiprobable, but the sample space $\{HH, HT, TH, TT\}$ is equiprobable. From this, we see that the probability of 0 heads, that is, $Pr(\{TT\})$ is $\frac{1}{4}$, the probability of 1 head is $Pr(\{HT, TH\}) = \frac{1}{2}$, and the probability of 2 heads is $\frac{1}{4}$.

Outcome	Probability
0 heads	$\frac{1}{4}$
1 head	$\frac{1}{2}$
2 heads	$\frac{1}{4}$