MATH 112, Section 001 Name Solutions
Spring, 2009
Exam 2

1. Using only OR and NOT gates, draw a circuit that accepts two inputs and outputs 0 if at least one input is 1 and outputs 1 otherwise. Be sure to label each gate that you use with either the word "OR" or the word "NOT".

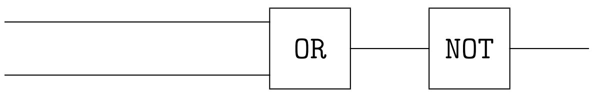
Thinking of 0 as FALSE and 1 as TRUE, we want a circuit which gives the truth table

T	T	F
T	F	F
F	T	F
F	F	T

Since the truth table for $p \vee q$ is

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

we want a circuit that corresponds to $\sim (p \vee q)$. Such a circuit is



2. Let $A = \{R, S, T, U, V\}$ and let $B = \{T, V, W, X, Y, Z\}$.

   (a) Find $A \cup B$.

   $A \cup B =$ $\{R, S, T, U, V, W, X, Y, Z\}$

   (b) Find $A \cap B$.

   $A \cap B =$ $\{T, V\}$

3. Let $A = \{1, 2, 3, 4, 5, 6, \}$, $B = \{2, 4, 6, 8\}$ and $C = \{5, 6, 7\}$.

   (a) Find $(A \setminus B) \cup C$

   $(A \setminus B) \cup C =$ $\{1, 3, 5, 6, 7\}$

   (b) Find $A \setminus (B \cup C)$

   $A \setminus (B \cup C) =$ $\{1, 3\}$

4. Suppose that A and B are sets such that $n(A) = 25$, $n(B) = 40$, and $n(A \cup B) = 50$.

   (a) Find $n(A \cap B)$

   `By the Inclusion-exclusion principle,` $n(A \cup B) = n(A) + n(B) - n(A \cap B)$`, so` $50 = 25 + 40 - n(A \cap B)$`. Therefore,` $n(A \cap B) = 25 + 40 - 50 = 15$`.`

   $n(A \cap B) =$ 15

(b) How many elements of $A \cup B$ are in exactly one of the sets A or B?

`The elements which are in exactly one of the sets` A `or` B `are the elements of` $(A \cup B) \setminus (A \cap B)$`. Since there are` 50 `elements in` $A \cup B$ `and` 15 `elements of` $A \cap B$`, there are` $50 - 15 = 35$ `elements that are in exactly one of the sets.`

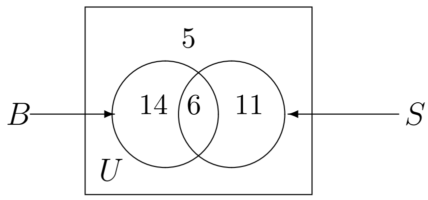
Answer:

35

5. In a particular class with 36 students, 20 students have at least one brother, 17 students have at least one sister, and 6 students have both a brother and a sister.

(a) How many students have neither a brother nor a sister?

`We draw a Venn diagram, with` B `being the set of students having a brother,` S `being the set of students having a sister, and the universe` U `being the set of students in the class.`



`Since` 5 `people are in neither circle,` 5 `students have neither a brother nor a sister.`

Answer:

5

(b) How many students have a brother but not a sister?

(There are 14 students in the region corresponding to $B \setminus S$, so 14 students have a brother but not a sister.

Answer:
14

6. An automobile dealership has 60 cars on the lot. Of these cars:

30 have GPS.

35 have satellite radio.

25 have pinstriping package.

15 have both GPS and satellite radio.

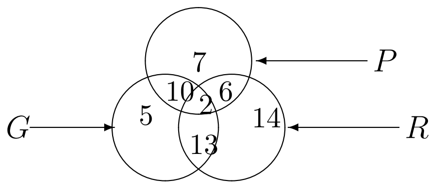
12 have both GPS and the pinstriping package.

8 have both satellite radio and the pinstriping package.

2 have all three options.

How many cars have at least one of the three options?

We make a Venn diagram with G being the set of cars with GPS, R being the set of cars with satellite radio, and P being the set of cars with the pinstriping package.



Adding the numbers in the circles we get 57 elements in $G \cup R \cup P$.

Answer:
57

7. How many ways are there of selecting a dinner of one appetizer, one entrée, one beverage, and one dessert if there are 4 choices for an appetizer, 6 choices for an entrée, 5 choices for a beverage, and 3 choices for a dessert?

   From the multiplication principle, the number of ways of selecting a dinner is $4 \cdot 6 \cdot 5 \cdot 3 = 360$.

Answer:
360

8. (a) Evaluate $P(6,2)$.

   $P(6,2) = 6 \cdot 5 = 30$.

$P(6,2) =$ 30

   (b) Evaluate $C(8,3)$.

   $C(8,3) = \frac{8!}{3!5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$.

$C(8,3) =$ 56

9. How many ways are there of choosing a president, vice-president, secretary, and treasurer from a group of 11 people?

   We are asking for the number of ways of choosing and arranging 4 things from a set of 11 distinguishable things. There are $P(11,4) = 11 \cdot 10 \cdot 9 \cdot 8 = 7920$ ways of doing that.

Answer:
7920

10. How many ways are there of choosing a four-person committee from a group of 11 people?

We are asking for the number of ways of choosing 4 things from a set of 11 distinguishable things. There are $C(11,4) = \frac{11!}{4!7!} = \frac{11\cdot 10\cdot 9\cdot 8}{4\cdot 3\cdot 2\cdot 1} = 330$ ways of doing that.

Answer:
330