MATH 112, Section 001 Name **Solutions**
Spring, 2009
Exam 1

1. Let p be the statement "Justin visits," let q be the statement "Mary leaves the country," and let r be the statement "Mary locks herself in her room." Write the statement "If Justin visits, then Mary will either leave the country or lock herself in her room" using the simple statements p, q, and r and logical connectives.

   Answer:
   $$p \to (q \vee r)$$

2. Let p be the statement "31 is a factor of 2293," let q be the statement "32 is a factor of 3007," and let r be the statement "31 is a factor of 4586." Write the statement $p \to ((\sim q) \wedge r)$ as an English sentence.

   Answer: `If 31 is a factor of 2293, then 32 is not factor of 3007 and 31 is a factor of 4586.`

In problems 3 through 5, make a truth table for the given statement form.

3. $(p \wedge q) \to (\sim p)$.

p	q	$p \wedge q$	$\sim p$	$(p \wedge q) \to (\sim p)$
T	T	T	F	F
T	F	F	F	T
F	T	F	F	T
F	F	F	T	T

4. $p \wedge (q \to (\sim p))$.

p	q	$\sim p$	$q \to (\sim p)$	$p \wedge (q \to (\sim p))$
T	T	F	F	F
T	F	F	T	T
F	T	T	T	F
F	F	T	T	F

5. $(p \vee q) \to (r \leftrightarrow (\sim q))$

p	q	r	$p \vee q$	$r \leftrightarrow (\sim q)$	$(p \vee q) \to (r \leftrightarrow (\sim q))$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	F	T	T
F	F	F	F	F	T

6. Write the contrapositive and the converse of the statement "If $2^{50} > 3^{36}$, then $4^{10} > 9^7$.

   `Notice that to say that it is not the case that` $2^{50} > 3^{36}$ `is to say that` $2^{50} \leq 3^{36}$. `A similar statement holds for the other inequality.`

   Contrapositive: `If` $4^{10} \leq 9^7$ `then` $2^{50} \leq 3^{36}$.

   Converse: `If` $4^{10} > 9^7$ `then` $2^{50} > 3^{36}$.

7. Determine whether or not the statement forms $\sim (p \wedge (\sim q))$ and $p \to q$ are logically equivalent. Be sure that you make clear how you arrive at your answer.

   `The truth table for` $\sim (p \wedge (\sim q))$ `is:`

p	q	$\sim q$	$p \wedge (\sim q)$	$\sim (p \wedge (\sim q))$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

The truth table for $p \to q$ is

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Since the last columns of the truth tables are the same, the statement forms are logically equivalent.

8. Determine whether or not the statement form $\sim ((\sim p) \wedge q)$ logically implies $p \wedge q$. Be sure that you make clear how you arrive at your answer.

We make a truth table for $\sim ((\sim p) \wedge q) \to (p \wedge q)$

p	q	$\sim p$	$(\sim p) \wedge q$	$\sim ((\sim p) \wedge q)$	$p \wedge q$	$\sim ((\sim p) \wedge q) \to (p \wedge q)$
T	T	F	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	F	F	T
F	F	T	F	T	F	F

Since $\sim ((\sim p) \wedge q) \to (p \wedge q)$ is not a tautology, $\sim ((\sim p) \wedge q)$ does not logically imply $p \wedge q$.

9. Let the universe consist of the set of all integers. Write the negation of the statement "$(\forall x)$ if 3 is a factor of x, then 9 is a factor of x." *Do not simply put "It is not the case that" or $\sim$ or something similar at the beginning of the sentence. The point here is to write the negation in a non-trivial way.*

   `The statement has the form` $(\forall x)p(x) \to q(x)$`. (In this case,` $p(x)$ `is the predicate ''3 is a factor of` x`'' and` $q(x)$ `is the predicate ''9 is a factor of` x`.'') The negation of such a statement is` $(\exists x) \sim (p(x) \to q(x))$`. Since` $\sim (p \to q)$ `is logically equivalent to` $p \wedge \sim q$`, the negation of the given statement is ''`$(\exists x)$ `3 is a factor of` x `but 9 is not a factor of` x`.''`

Answer:
$(\exists x)$ `3 is a factor of` x `but 9 is not a factor of` x`.`

10. Determine whether or not the following argument is logically valid. Explain your answer fully.

    If John buys a lottery ticket, and if he wins, then he will buy a gvantzenkvetcher.

    John is not buying a gvantzenkvetcher.

    Therefore, John did not buy a lottery ticket.

    ☐ The argument is logically valid.

    ☒ The argument is not logically valid.

Explanation:

Let p be the statement ''John buys a lottery ticket,'' q be the statement ''John wins'', and r be the statement, ''John buys a gvantzenkvetcher.'' The argument is $((p \wedge q) \rightarrow r) \wedge (\sim r)$ implies $\sim p$. Since the implication $(((p \wedge q) \rightarrow r) \wedge (\sim r)) \rightarrow \sim p$ is false when p is true and q and r are false, this argument is not logically valid.